Predicting Diffractive ρ and φ Production using Light-Front Holographic Wavefunctions

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- Collider Physics

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- The Vector Meson
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Introduction

- **Purpose:** to predict vector meson production cross sections for φ mesons by modelling the φ meson with a **light-front holographic wavefunction**.
- How can elementary particles be detected?
- Use collision experiments where particles are collided into one another and the results are observed.
- Infer properties from conservation laws.
- For example: deep inelastic scattering

Introduction

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Diffractive ρ and ϕ production at HERA using a holographic AdS/QCD light-front meson wave function

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We use an anti–de Sitter/quantum chromodynamics holographic light-front wave function for the ρ and ϕ mesons, in conjunction with the color glass condensate dipole cross section whose parameters are fitted to the most recent 2015 high precision HERA data on inclusive deep inelastic scattering, in order to predict the cross sections for diffractive ρ and ϕ electroproduction. Our results suggest that the holographic meson light-front wave function is able to give a simultaneous description of ρ and ϕ production data provided we use a set of light quark masses with $m_{u,d} < m_s \approx 0.14$ GeV.

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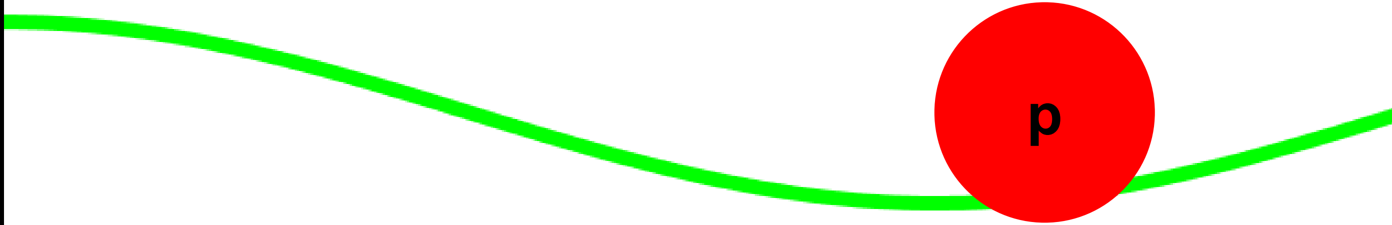
Introduction

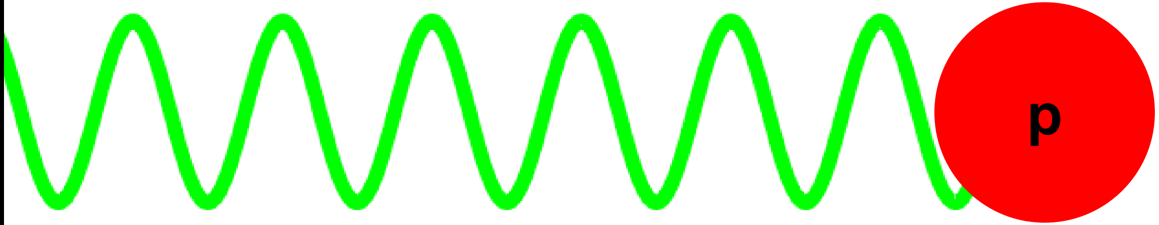
- If the electron's de Broglie wavelength is too long, it will not resolve the proton.



$$\lambda = \frac{h}{p}$$

Introduction

- By increasing the momentum, the de Broglie wavelength will decrease, making the proton resolvable. Therefore, high energies are needed for collision experiments.



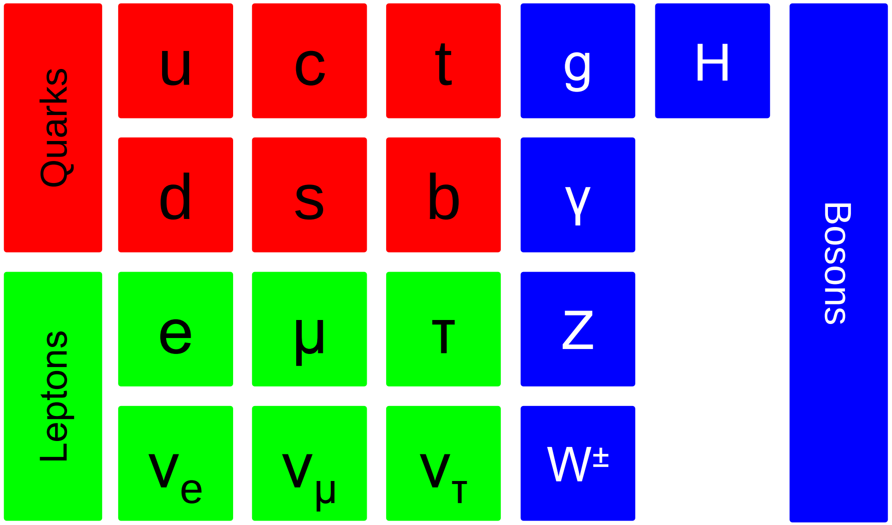
$$\lambda = \frac{h}{p}$$

The Standard Model

- The model used to describe elementary particle physics
- Three fundamental forces:
  - The electromagnetic force (quantum electrodynamics)
  - The strong force (quantum chromodynamics)
  - The weak force (weak interactions)
- Two types of particles:
  - Fermions
  - Bosons

The Standard Model

- Bosons mediate forces (aside from the Higgs boson).
- Fermions are further divided into quarks and leptons:



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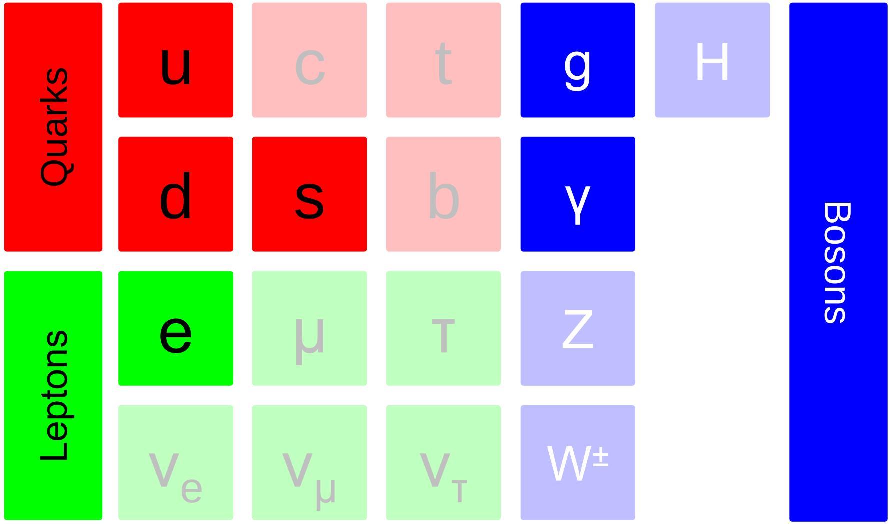
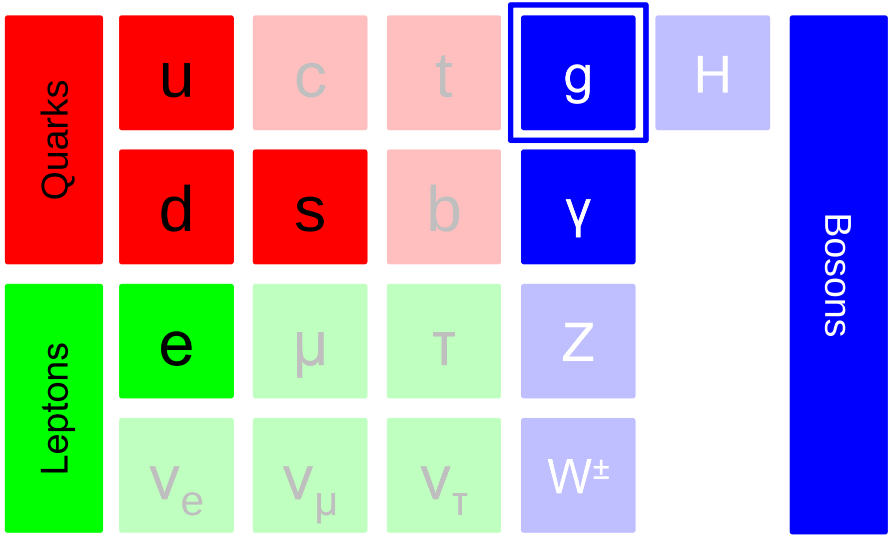


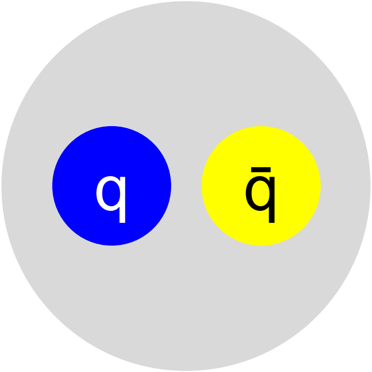
Quantum Chromodynamics

- Quarks can interact via the strong force, mediated by gluons. Ascribe a property called colour charge to every quark and gluon.

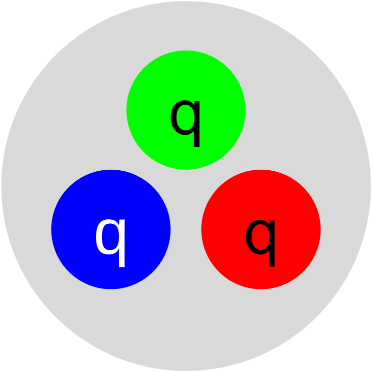


Quantum Chromodynamics

- Quarks can be red, green, or blue.
- Only colour singlet states have been observed in nature. Quarks can bind to other quarks in two ways to form colour singlet states:



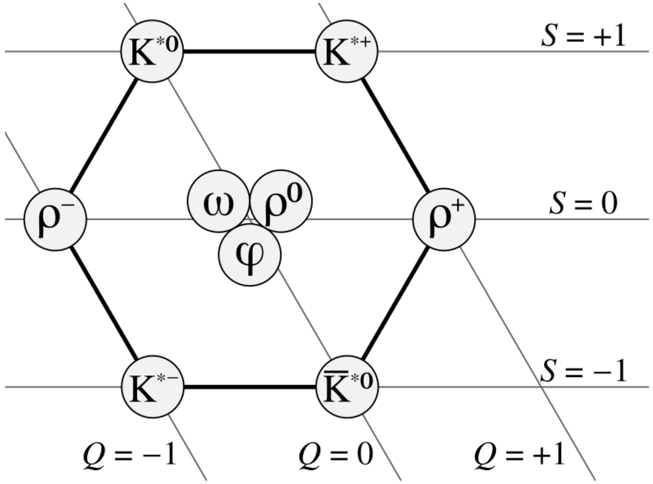
Mesons



Baryons

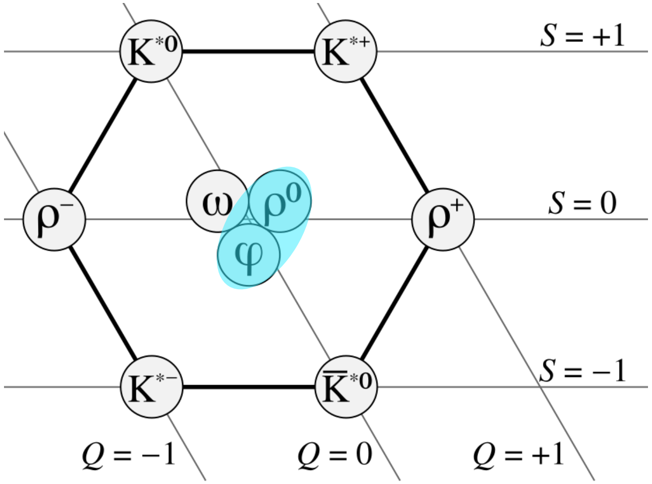
Vector Mesons

- Vector mesons are mesons with spin 1 and parity -1 (they transform like vectors under parity transformations).



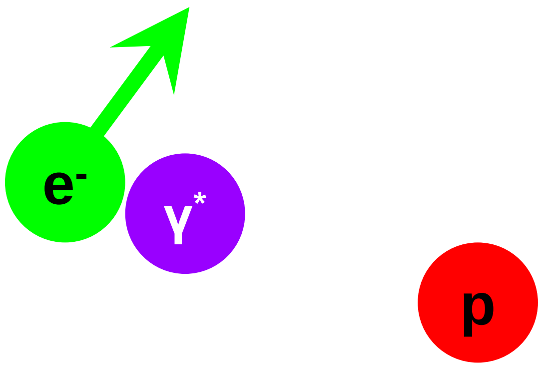
Vector Mesons

- ρ^0 mesons are a superposition of $u\bar{u}$ and $d\bar{d}$.
- φ mesons are $s\bar{s}$.



Collider Physics

Collider Physics



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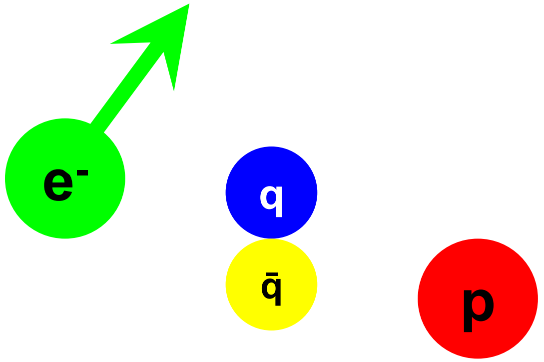

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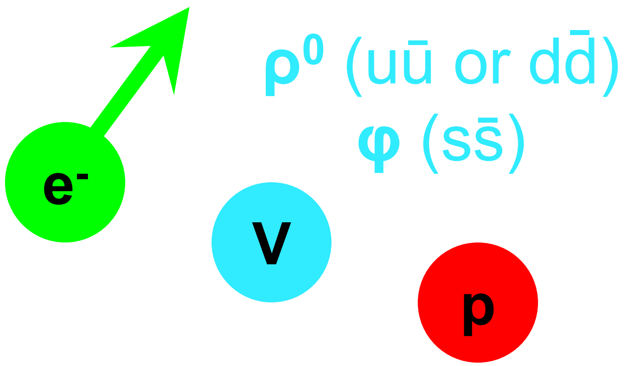
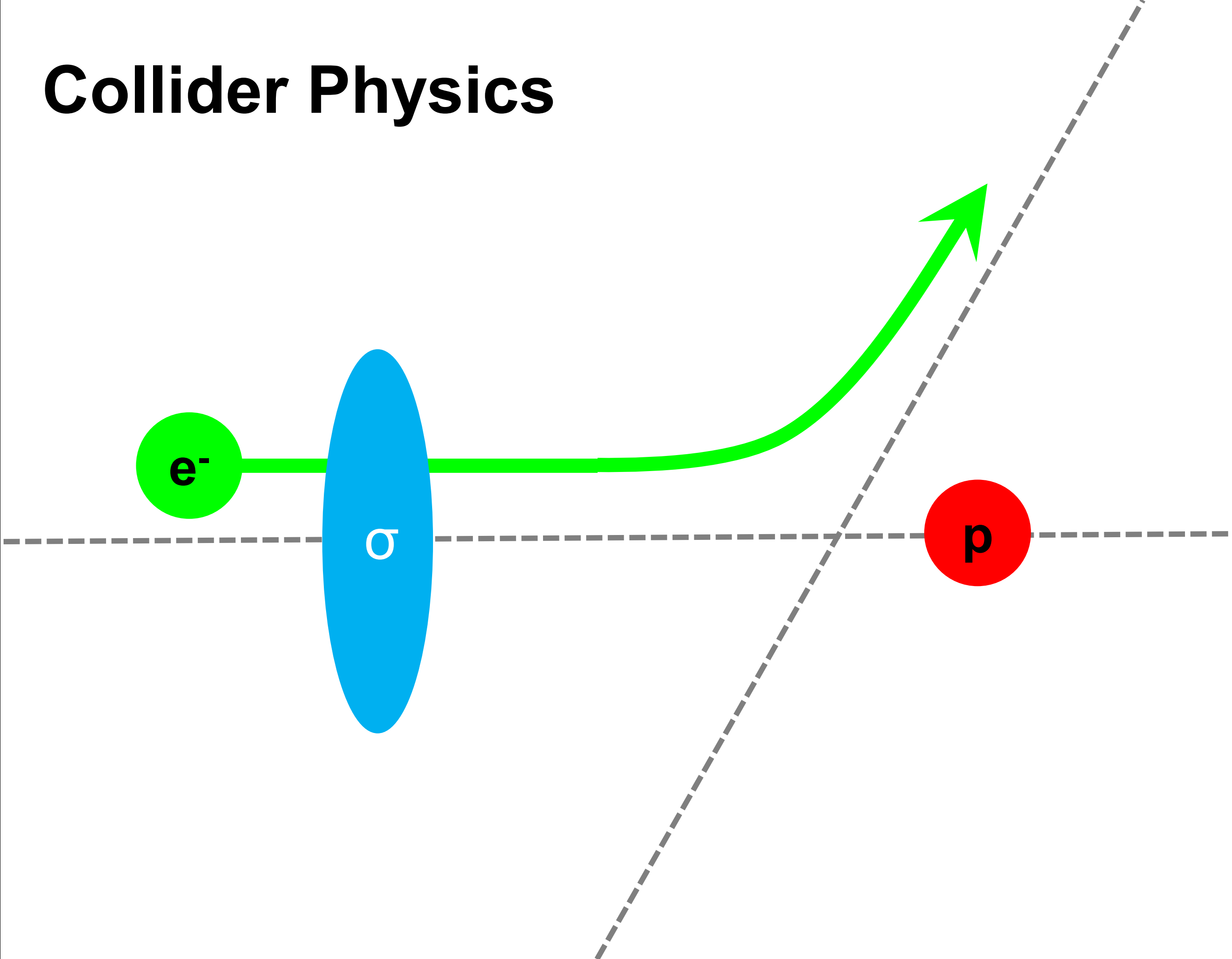
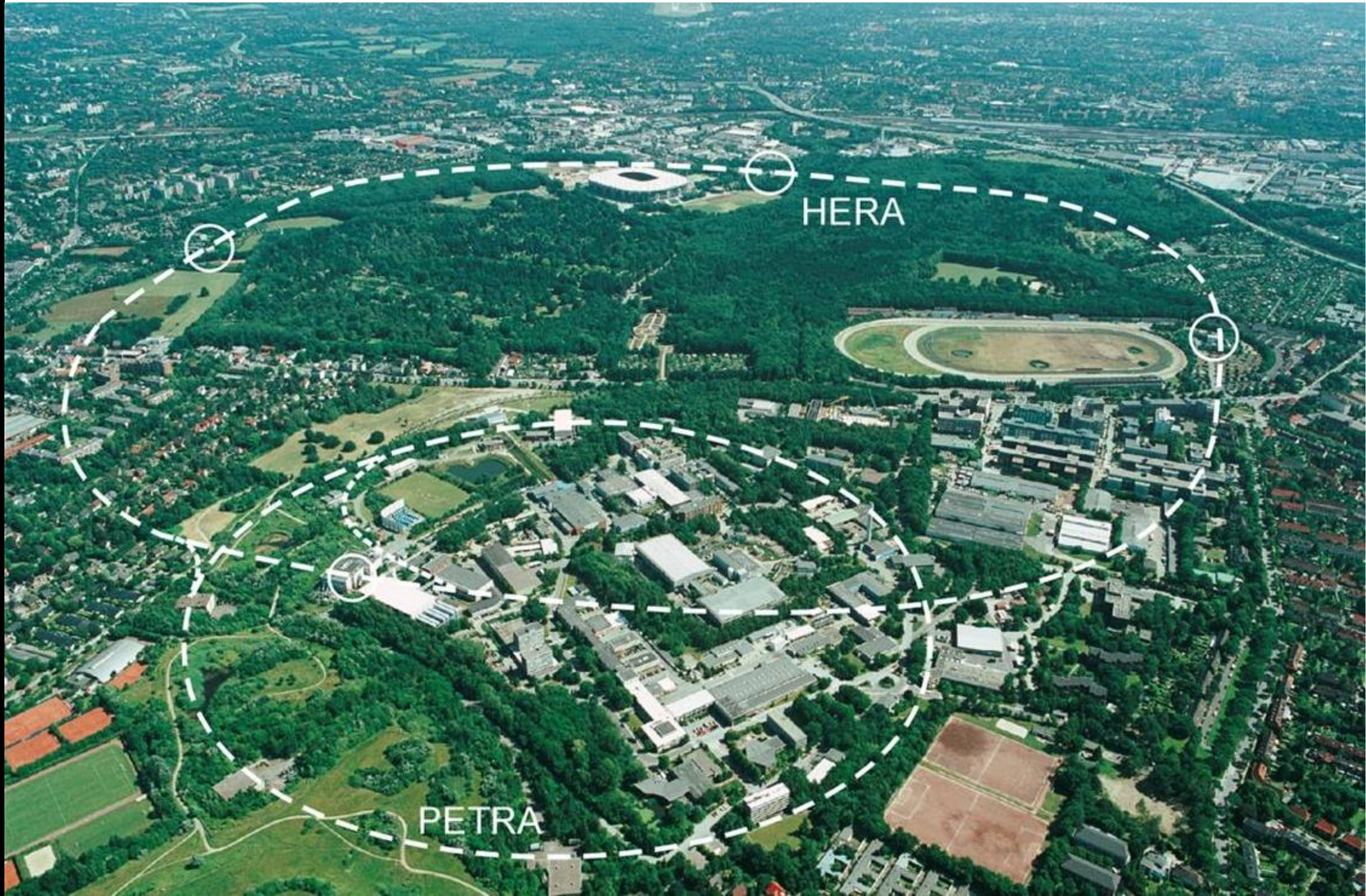


Collider Physics



The HERA Experiments



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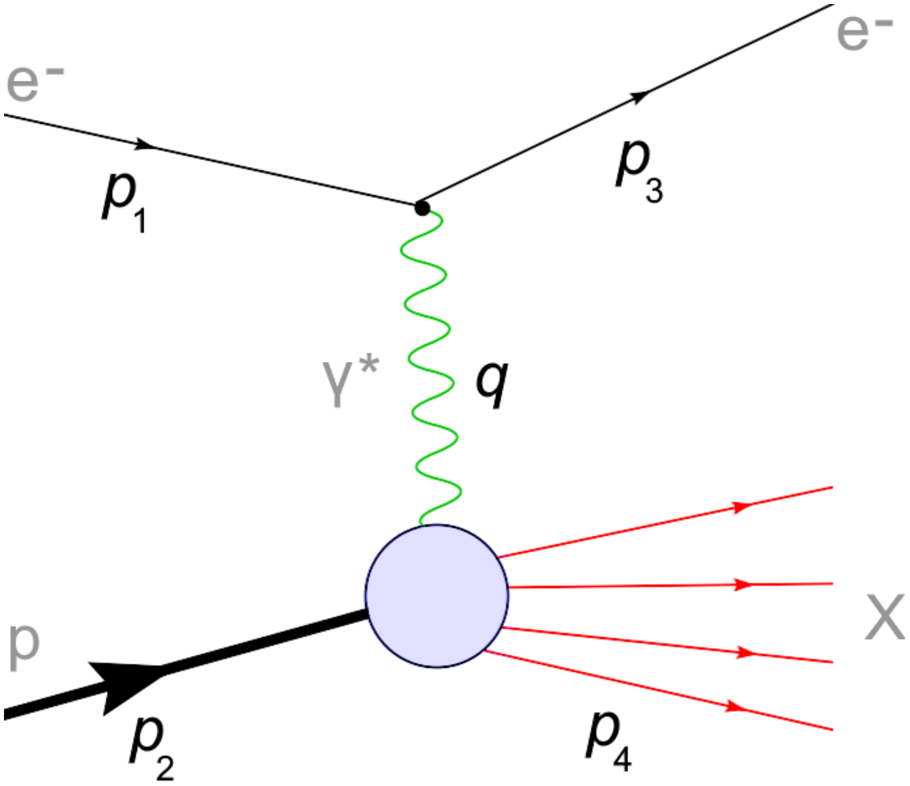
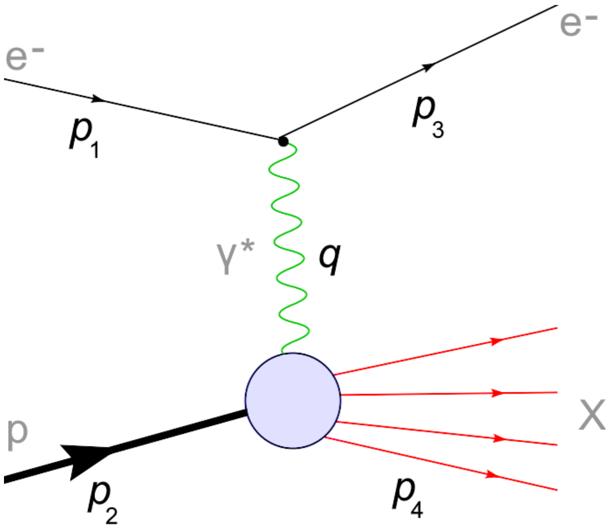


The HERA Experiments

- Define important kinematic variables:



$$Q^2 = -q^2$$

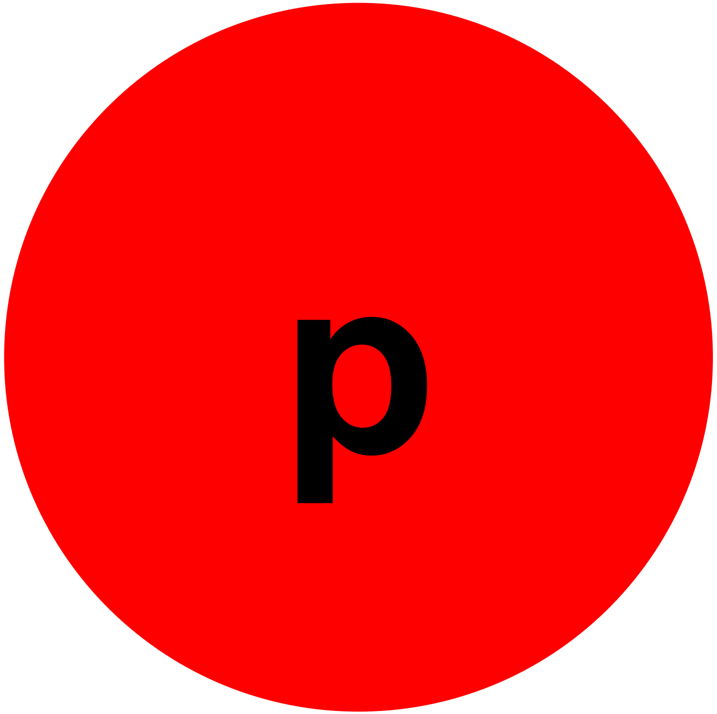
$$x_{Bj} = \frac{Q^2}{2\underline{p}_2 \cdot \underline{q}}$$

$$W = \sqrt{\frac{Q^2}{x_{Bj}}}$$

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$

$$t = (p_1 - p_3)^2 = (p_4 - p_2)^2$$

The Proton



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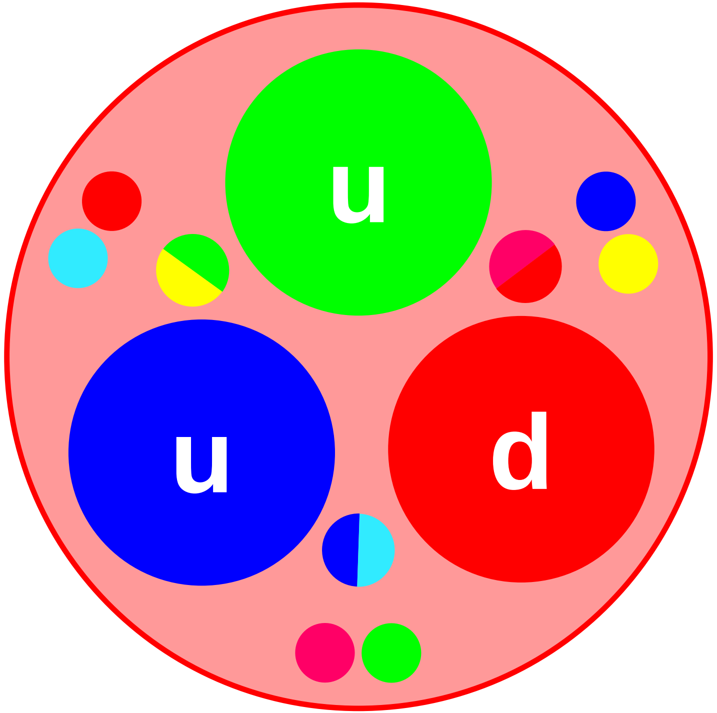
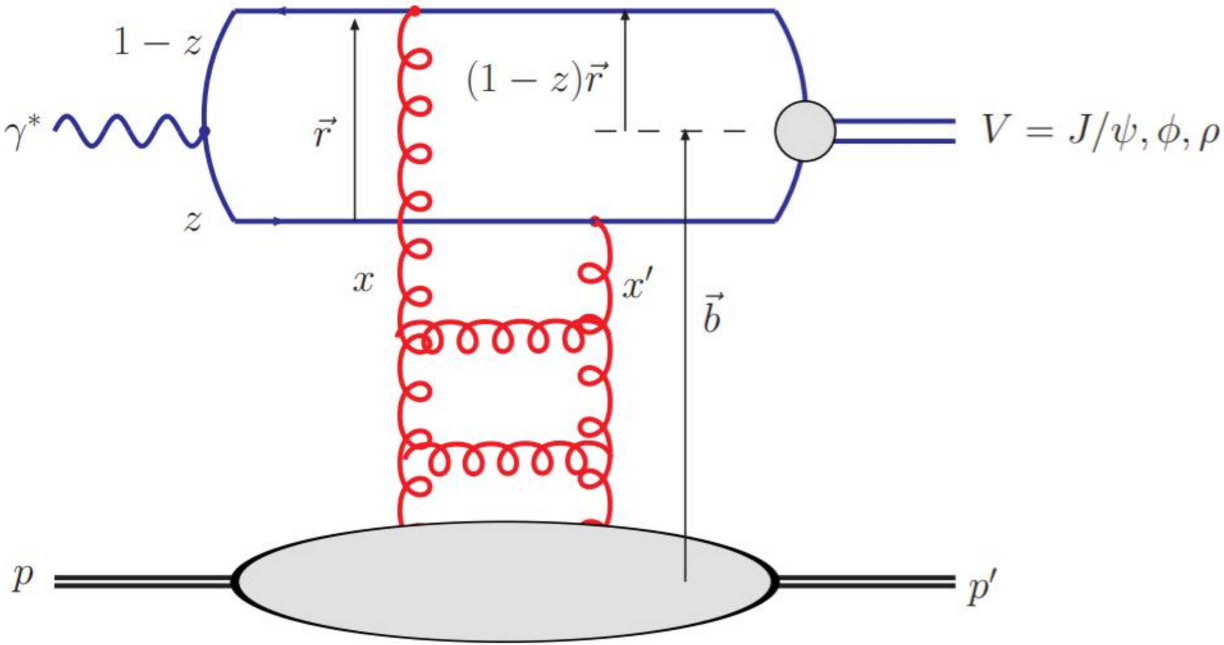


Modelling the Reaction



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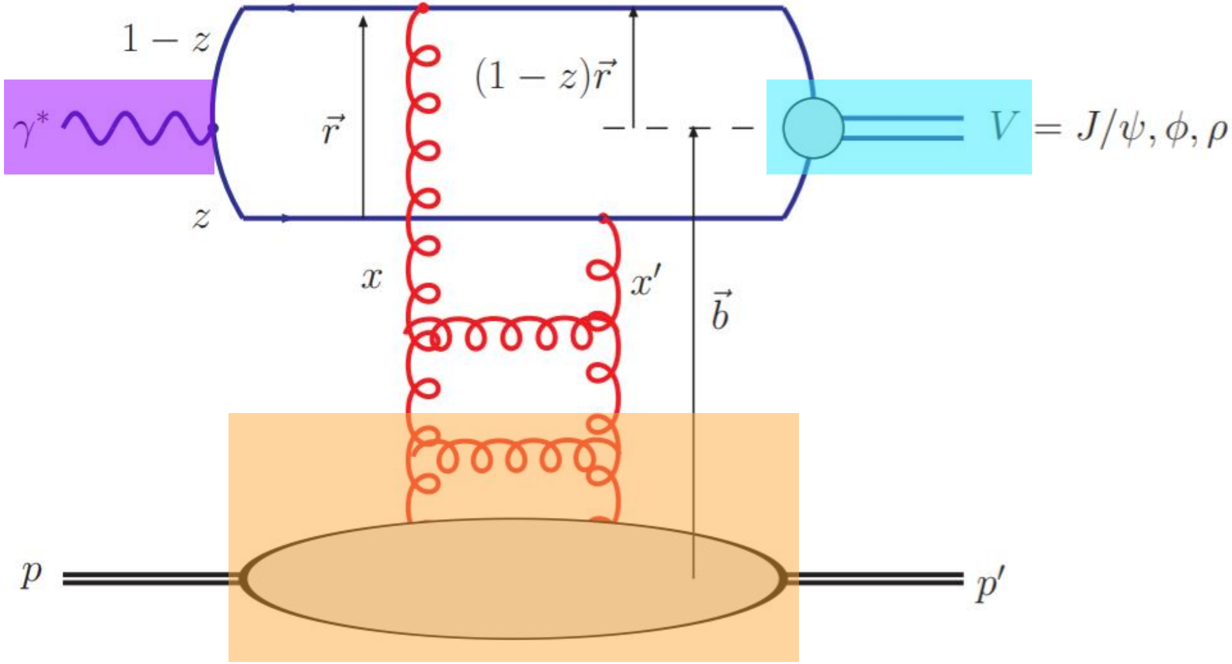

The Vector Meson Wavefunction

Use light-front dynamics and its holographic mapping to gravity in a higher-dimensional anti-de Sitter (AdS) space to get a relativistic light-front wave equation for arbitrary spin (Brodsky, et al., 2015).

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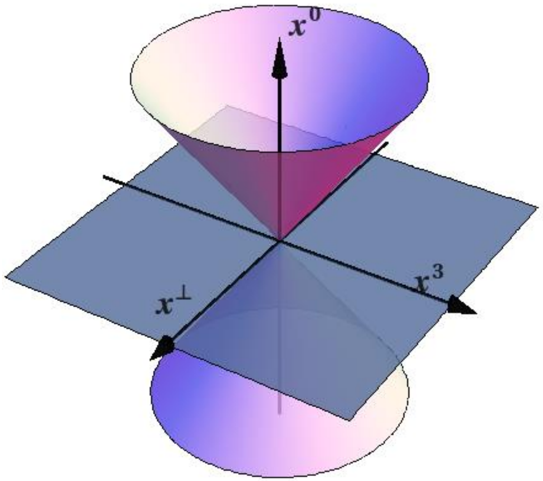
Light-Front Dynamics



- From relativistic dynamics
- Use the light-front form of wavefunction, rather than the instant form
- Represented as the plane on the edge of the light cone

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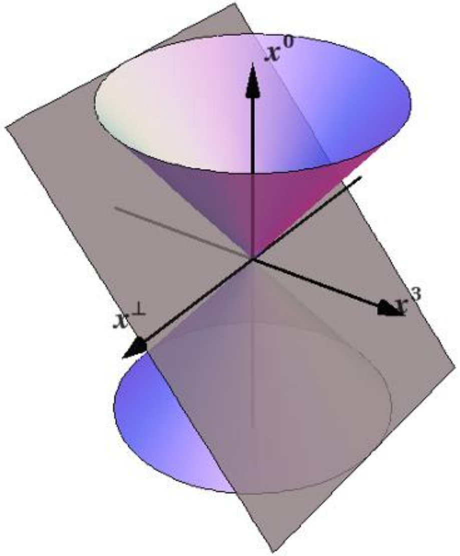
Light-Front Dynamics



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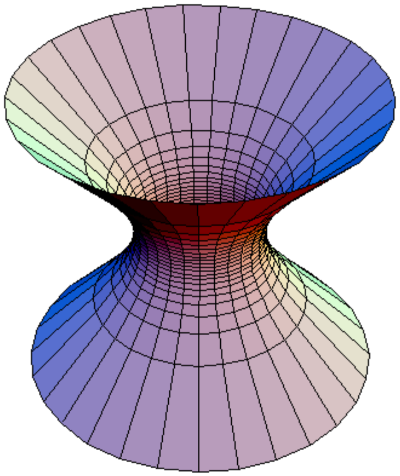
Holographic Mapping

- If a quantum theory in one space, of dimension *d*, corresponds to a gravitational theory in another space, of dimension $d + 1$, they are holographic duals.
- Mappings can be defined to go from one to the other.
- Strong interactions in 4D and weak interactions in 5D are approximately holographic duals.

The Vector Meson Wavefunction

Use light-front dynamics and its holographic mapping to gravity in a higher-dimensional anti-de Sitter (AdS) space to get a relativistic light-front wave equation for arbitrary spin (Brodsky, et al., 2015).

Anti-de Sitter Space



- A maximally-symmetric Lorentzian manifold with constant negative curvature.
- AdS/QCD mapping
- Work in five-dimensional anti-de Sitter space (one temporal and four spatial dimensions)

The Vector Meson Wavefunction

- By writing an Eigenvalue equation where the square of the vector meson's mass are the Eigenvalues:

$$H_{LF}\left|\psi\right\rangle = M_V^2\left|\psi\right\rangle$$

- The holographic Light-Front Schrödinger Equation can be derived:

$$\left(-\frac{\mathrm{d}^2}{\mathrm{d}\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = M_V^2\phi(\zeta)$$

The Vector Meson Wavefunction

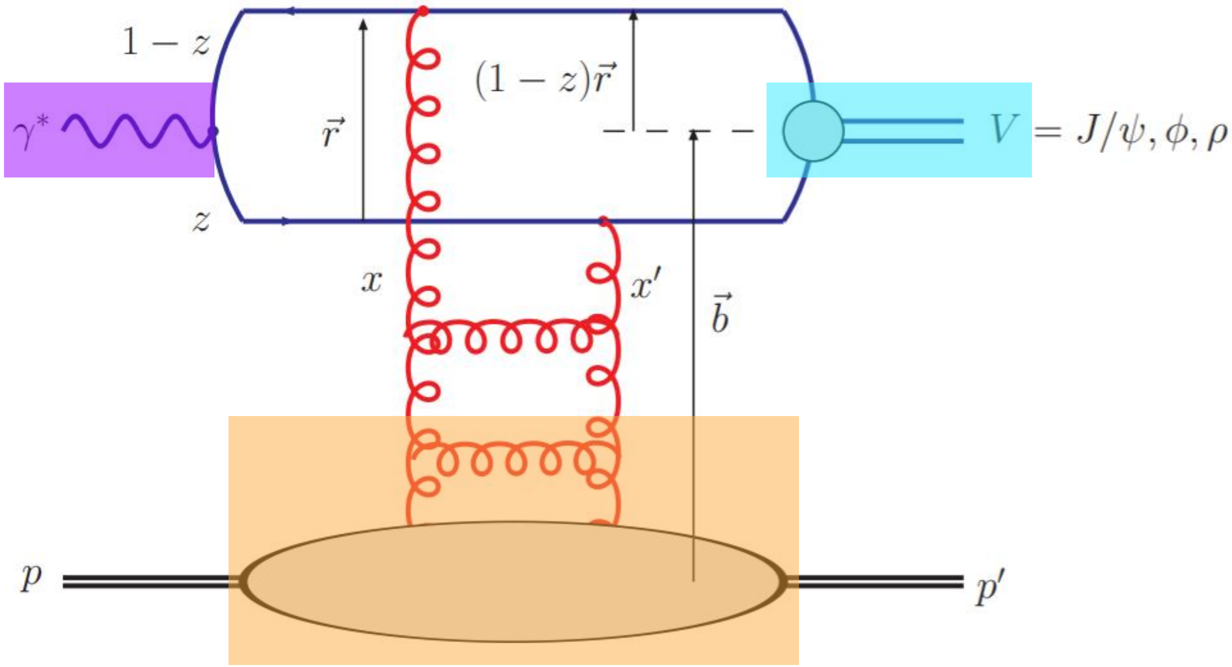
- From the solutions to the holographic SE, longitudinal and transverse wavefunctions can be written for the vector meson (Brodsky, et al., 2015).

$$\Psi_\lambda(x,\zeta) = N_\lambda \sqrt{x(1-x)} e^{-\frac{\kappa^2\zeta^2}{2}} e^{-\frac{m_f^2}{2\kappa^2 x(1-x)}}$$

$$\Psi^{V,L}_{h,\bar{h}}(x,r) = \frac{1}{2}\delta_{h,\bar{h}}\left(1 + \frac{m_f^2 - \nabla_r^2}{x(1-x)M_V^2}\right)\Psi_L(x,r)$$

$$\Psi^{V,T}_{h,\bar{h}}(x,r) = \pm\left(ie^{\pm i\theta_r}\left(x\delta_{h\pm,\bar{h}\mp} - (1-x)\delta_{h\mp,\bar{h}\pm}\right)\partial_r + m_f\delta_{h\pm,\bar{h}\pm}\right)\frac{\Psi_T(x,r)}{2x(1-x)}$$

Modelling the Reaction



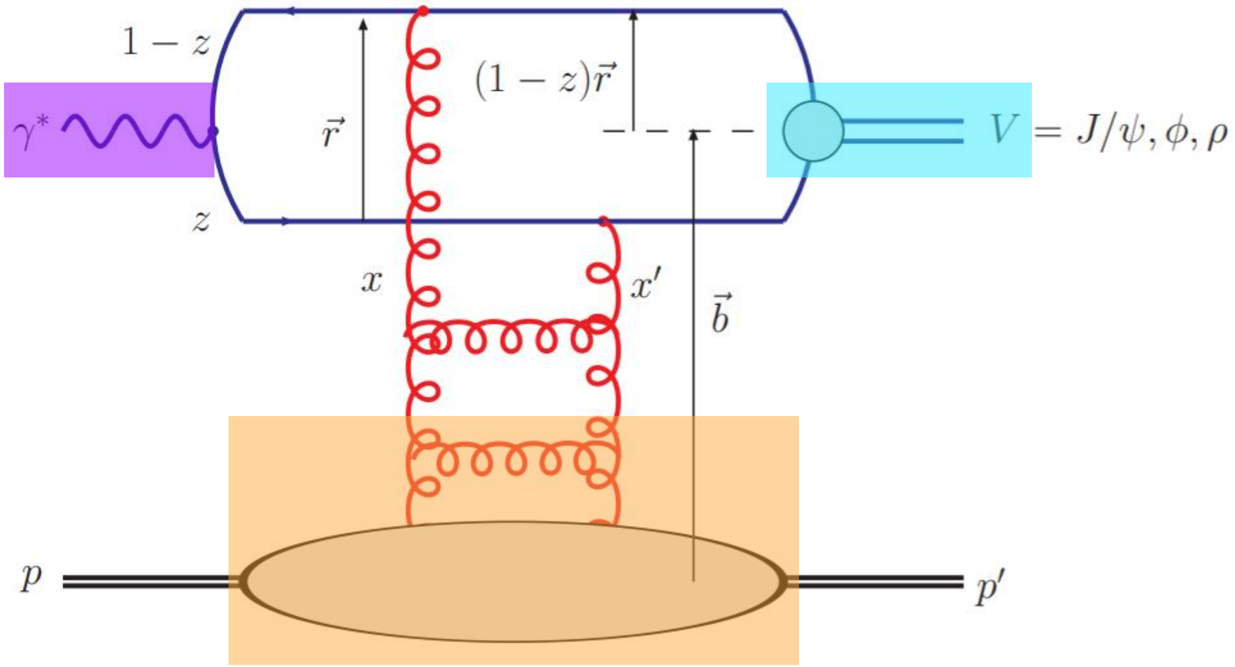
The Photon Wavefunction

- The photon is a point particle, and there are no strong interactions involved.
- Therefore, use light-front perturbation theory.

$$\Psi^{\gamma,L}_{h,\bar{h}}(x,r;Q^2) = \sqrt{\frac{N_c}{4\pi}}(\delta_{h,-\bar{h}} e e_f)\left(2x(1-x)\sqrt{Q^2}\right)\frac{\mathrm{K}_0(\varepsilon r)}{2\pi}$$

$$\Psi^{\gamma,T}_{h,\bar{h}}(x,r;Q^2) = \pm\sqrt{\frac{N_c}{2\pi}} e e_f \left(i e^{\pm i\theta_r}(x\delta_{h\pm,\bar{h}\mp} - (1-x)\delta_{h\mp,\bar{h}\pm})\partial_r + m_f \delta_{h\pm,\bar{h}\pm}\right)\frac{\mathrm{K}_0(\varepsilon r)}{2\pi}$$

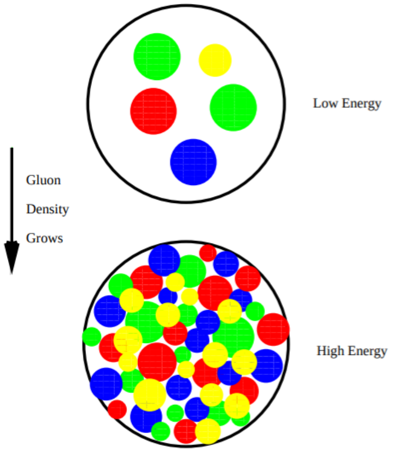
Modelling the Reaction



The Dipole-Proton Interaction

- Use the colour glass condensate (CGC) model.
  - Colour: Gluons have colour charge
  - Glass: The gluons fields evolve slowly over long timescales
  - Condensate: The density of gluons is very high

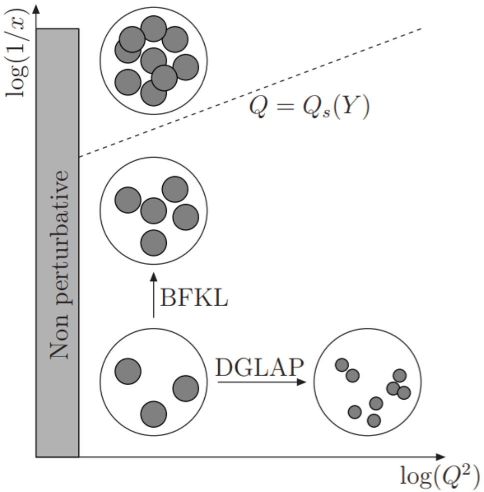
- Gluon density grows as energy increases.



The Dipole-Proton Interaction

- Define the saturation scale, Q_s, as the point at which the gluon density is saturated, and no longer increases.

$$Q_s(x_{Bj}) = \left(\frac{x_0}{x_{Bj}}\right)^{\frac{\lambda}{2}}$$



- Perturbative → non-perturbative as x approaches the saturation region.
- Balitsky-Fadin-Kuraev-Lipatov (BFKL) and Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) cannot describe the evolution.

The Dipole-Proton Interaction

- Write the cross section in terms of a parameter σ_0 and a scattering amplitude, *N*:

$$\hat{\sigma}_{q\bar{q}}^{CGC}(x,r) = 2\int \mathrm{d}^2\underline{b}\mathcal{N}(x,r,b) = \sigma_0\mathcal{N}(x,r)$$

$$\mathcal{N}(x,r) = \begin{cases} \mathcal{N}_0\left(\frac{rQ_s}{2}\right)^{2\left(\gamma_s + \frac{1}{\kappa\lambda\ln\left(\frac{1}{x}\right)}\ln\left(\frac{2}{rQ_s}\right)\right)} & rQ_s \leq 2 \\ 1 - e^{-A\ln^2(BrQ_s)} & rQ_s > 2 \end{cases}$$

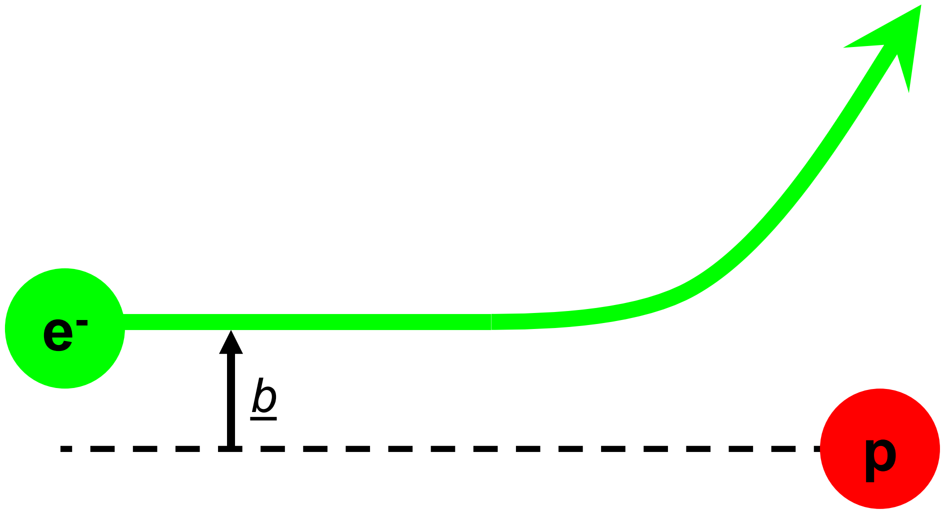
The Dipole-Proton Interaction

- The CGC model has four free parameters.
- These must be found before doing any calculations.
- Find them by fitting to the 2015 HERA data.

$$\sigma_0, \quad x_0, \quad \gamma_s, \quad \lambda$$

Impact-Parameter Dependence

- Dependence on the impact-parameter $\underline{b}$ was introduced by H. Kowalski, L. Motyka, and G. Watt in 2006.



Impact-Parameter Dependence

- This is the **b-CGC model**.
- The dependence is expressed as an exponent in the saturation scale (Watt & Kowalski, 2008).

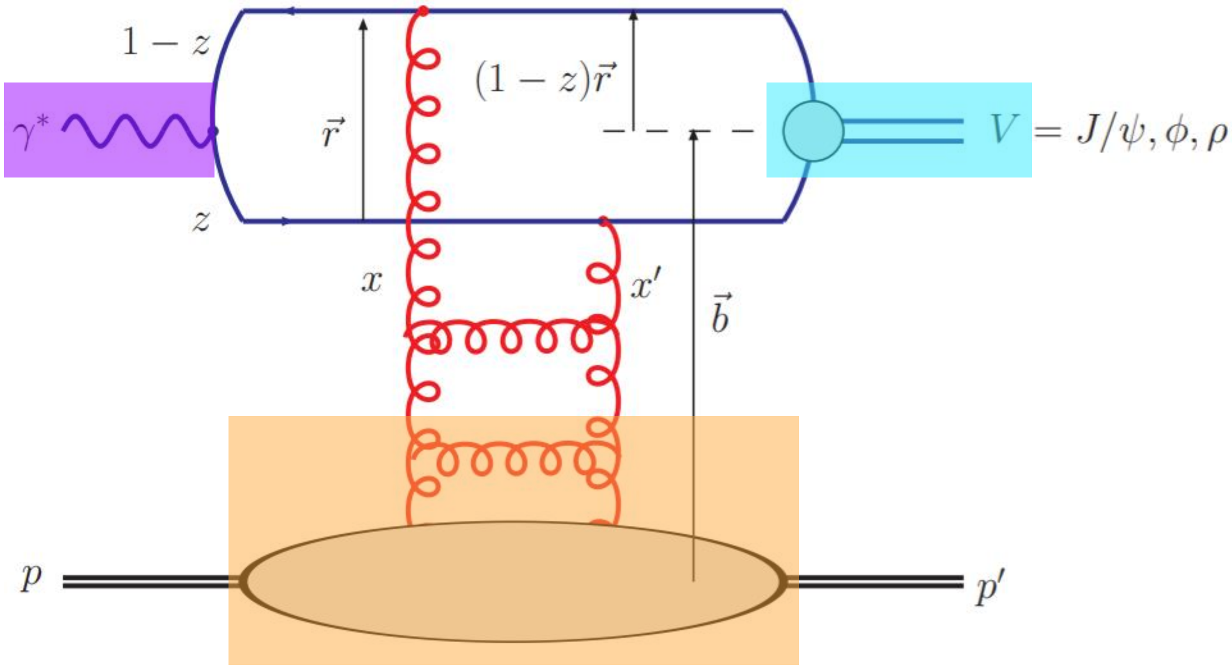
$$Q_s(x_{Bj}, b) = \left(\frac{x_0}{x_{Bj}}\right)^{\frac{\lambda}{2}} e^{-\frac{b^2}{4\gamma_s B_{CGC}}}$$

$$\hat{\sigma}_{q\bar{q}}^{b-CGC} = \int \mathrm{d}^2\underline{b}\; 2\mathcal{N}(x, r, b)$$

- b-CGC has five free parameters:

$$x_0, \quad \gamma_s, \quad \lambda, \quad B_{CGC}, \quad \mathcal{N}_0$$

Modelling the Reaction



The Physical Observables

- Multiply the three parts of the interaction and integrate over kinematic variables to calculate the imaginary part of the scattering amplitude.

- For the CGC model:

$$\mathcal{I}\mathrm{m}\mathcal{A}_\lambda(s;Q^2) = \sum_{h,\bar{h}} \int \mathrm{d}^2\underline{r}\mathrm{d}x\, \underbrace{\Psi^{\gamma^*,\lambda}_{h,\bar{h}}(x,r;Q^2)}_{\substack{\text{Light-front}\\ \text{perturbation theory}}} \underbrace{\Psi^{V,\lambda}_{h,\bar{h}}(x,r)^*}_{\substack{\text{Light-front}\\ \text{holography}}} \underbrace{\sigma_0\mathcal{N}(x,r)}_{\text{CGC}}$$

- For the b-CGC model:

$$\mathcal{I}\mathrm{m}\mathcal{A}_\lambda(s,t;Q^2) = \sum_{h,\bar{h}} \int \mathrm{d}^2\underline{r}\mathrm{d}x\, \underbrace{\Psi^{\gamma^*,\lambda}_{h,\bar{h}}(x,r;Q^2)}_{\substack{\text{Light-front}\\ \text{perturbation theory}}} \underbrace{\Psi^{V,\lambda}_{h,\bar{h}}(x,r)^*}_{\substack{\text{Light-front}\\ \text{holography}}} e^{-ix\underline{r}\cdot\Delta} \underbrace{\mathcal{N}(x,r,\Delta)}_{\text{QCD}}$$

Differential Cross Sections

- Square the scattering amplitude to calculate the differential cross section. Assume an exponential dependence on *t* for the CGC model:

$$\frac{\mathrm{d}\sigma_\lambda}{\mathrm{d}t} = \frac{1}{16\pi}\left|\mathcal{I}\mathrm{m}\mathcal{A}_\lambda(s;Q^2)\right|^2(1+\beta_\lambda^2)e^{-B_D t}$$

- Do not assume anything for the b-CGC model, since the *t*-dependence is being predicted.

$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}t} = \frac{1}{16\pi}\left|\mathcal{I}\mathrm{m}\mathcal{A}_\lambda(s,t;Q^2)\right|^2(1+\beta_\lambda^2)R_g^2(\alpha_\lambda)$$

Cross Sections

- Integrate over *t* to calculate the longitudinal and transverse cross sections.
- In the CGC model:

$$\sigma_\lambda = \frac{1}{16\pi} \left| \mathcal{I}\mathrm{m}\mathcal{A}_\lambda(s, Q^2) \right|^2 (1 + \beta_\lambda^2) \left(\frac{1}{B_D} \right)$$

- In the b-CGC model:

$$\sigma_\lambda = \int \left(\frac{\mathrm{d}\sigma_\lambda}{\mathrm{d}t} \right) \mathrm{d}t$$

The Total Cross Section

- Add the transverse and longitudinal cross sections to calculate the total cross section:

$$\sigma = \sigma_T + 0.98\sigma_L$$

- Define *R*, the ratio between cross section parts:

$$R = \frac{\sigma_L}{\sigma_T}$$

Python Fits

- Values must be found for the CGC and b-CGC model parameters by fitting to data.
- Use the 2015 HERA data (with $Q^2 \in [0.045, 45]$ GeV^2 and $x_{Bj} \leq 0.01$), and fit with a Python program.

Q^2 GeV^2	x_{Bj}	$\sigma^+_{r,\mathrm{NC}}$	δ_{stat} %	δ_{uncor} %	δ_{cor} %	δ_{rel} %	$\delta_{\gamma p}$ %	δ_{had} %	δ_1 %	δ_2 %	δ_3 %	δ_4 %	δ_{tot} %
0.15	0.502×10^{-5}	0.185	3.79	1.50	3.62	1.39	0.35	−0.21	−0.17	−0.01	0.00	0.01	5.65
0.2	0.669×10^{-5}	0.227	1.65	0.78	1.70	0.86	0.57	0.00	0.00	0.00	0.00	0.01	2.70
0.2	0.849×10^{-5}	0.223	1.61	0.61	2.19	1.06	0.55	−0.26	−0.06	0.00	0.00	0.00	3.04
0.2	0.110×10^{-4}	0.208	2.79	1.50	2.83	1.01	0.34	−0.08	−0.18	0.00	0.00	0.01	4.38
0.2	0.398×10^{-4}	0.211	14.93	11.96	5.18	0.33	4.70	2.93	1.57	−0.03	−0.02	0.15	20.64
0.2	0.251×10^{-3}	0.180	13.49	6.17	3.00	0.32	1.39	−1.67	1.19	0.01	0.02	0.02	15.34
0.25	0.836×10^{-5}	0.265	1.46	0.73	1.92	1.17	0.63	−0.23	0.45	0.00	0.00	0.01	2.89
0.25	0.106×10^{-4}	0.260	1.29	0.66	1.84	1.11	0.63	−0.10	0.32	0.00	0.00	0.01	2.69
0.25	0.138×10^{-4}	0.249	1.27	0.72	1.85	1.24	0.61	−0.22	0.08	0.00	0.00	0.00	2.74
0.25	0.230×10^{-4}	0.243	1.41	1.50	2.37	2.23	0.38	−0.60	0.43	0.00	−0.02	0.01	3.94
0.25	0.398×10^{-4}	0.236	3.32	1.54	2.79	0.50	1.03	0.29	0.21	0.00	0.01	0.02	4.76
0.25	0.110×10^{-3}	0.199	3.96	1.50	2.50	0.77	0.32	0.06	−0.58	0.00	0.00	0.01	5.02
0.25	0.251×10^{-3}	0.196	3.75	1.44	3.26	−0.23	0.35	0.51	−0.21	0.01	0.02	0.02	5.22

Python Fits

- Fitting is done with the `curve_fit()` function from the `scipy` library.
- The program is written for parallel computing on ACE-NET supercomputers.

```
def parallel(sQ2x, *p):

    nProcesses = os.environ.get('OMP_NUM_THREADS', default = 0)
    nProcesses = int(nProcesses)
    if nProcesses == 0:
       nProcesses = None

    s, Qsq, xBj = sQ2x
    pList = []
    for param in p:
        pList.append(repeat(param))

    # Create pool of cpus and distribute cross section calculations

    with mp.Pool(nProcesses) as pool:
        answer = pool.starmap(redsigma, zip(s, Qsq, xBj, *pList))

    return answer
```

Python Fits

- Light quark mass: $m_{u,d,s}$ = 0.14 GeV
- Charm quark mass: m_c = 1.27 GeV
- N_0 = 0.558

```
Best Fit Estimates:
BCGC: 6.51384545963 +/- 0.190047008786
x0: 9.67573820879e-06 +/- 2.98905926743e-06
gammas: 0.545418479496 +/- 0.00935223272521
lambda: 0.140655603202 +/- 0.0035384521905

Total Chi-Squared: 678.93399952
Degrees of Freedom: 520
Reduced Chi-Squared: 1.30564230677
P Value: 3.08505653052e-06
```

Python Fits

- Judge these results by doing a chi-square goodness of fit test.

$$\chi^2 = \sum_i \frac{(E_i - T_i)^2}{T_i}$$

- From χ^2, calculate the reduced χ^2.
- The reduced χ^2 should be close to 1.0.
- χ^2 = 1.3 indicates the fit is good.

Python Fits

- Run a second fit to a different data set with B_{CGC} fixed at 5.5 GeV^{-2}.

```
Best Fit Estimates:
gammas: 0.662002661277
N0: 1.69076714587e-14
x0: 0.0013285927118
lambda: 0.206606324714
```

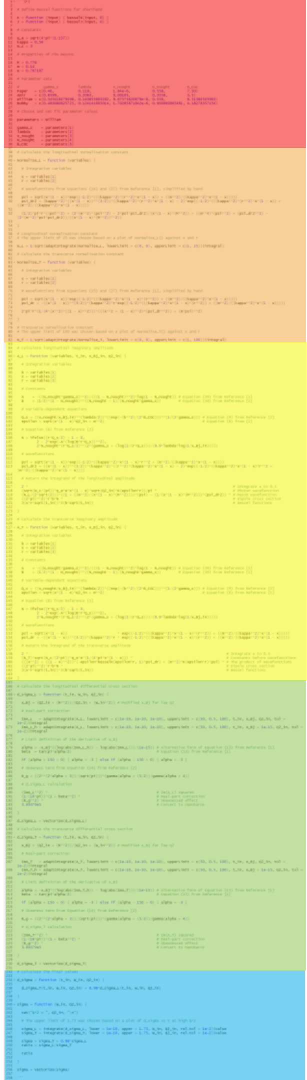
- Compare to Rezaeian, A. & Schmidt, I. (2013):

$B_{CGC}/\mathrm{GeV}^{-2}$	m_c/GeV	γ_s	N_0	x_0	λ	$\chi^2/\mathrm{d.o.f.}$
5.5	1.27	0.6599 ± 0.0003	0.3358 ± 0.0004	$0.00105 \pm 1.13 \times 10^{-5}$	0.2063 ± 0.0004	$368.4/297 = 1.241$
5.5	1.4	0.6492 ± 0.0003	0.3658 ± 0.0006	$0.00069 \pm 6.46 \times 10^{-6}$	0.2023 ± 0.0003	$370.9/297 = 1.249$

TABLE II: Parameters of the b-CGC dipole model, determined from fits to data in the range $x \leq 0.01$ and $Q^2 \in [0.75, 650]$ GeV^2. Results are shown for fixed light-quark masses $m_u = 10^{-2} \div 10^{-4}$ GeV and two fixed values of the charm quark masses (see the text for details).

R Program

- Define constants and set the fit parameter values.
- Calculate L and T normalisation constants for the meson wavefunction.
- Calculate imaginary L and T scattering amplitudes.
- Calculate L and T differential cross sections.
- Calculate and return the final values.



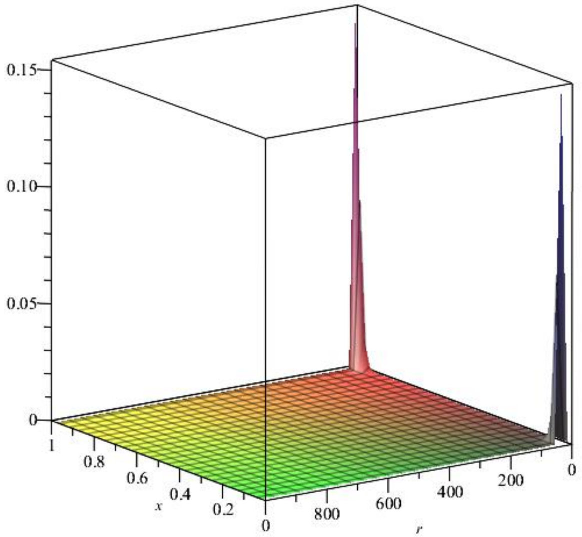
R Program

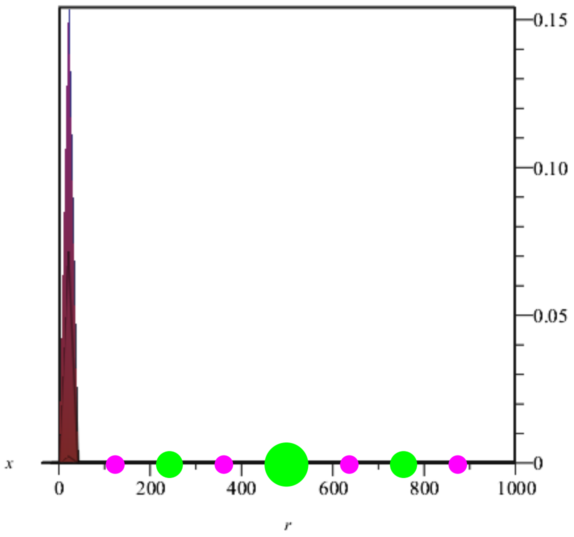
- Calculate integrals using the `adaptIntegrate()` function from the `cubature` package.
- Multidimensional adaptive integration over a hypercube.

```
Normalise_L = function (variables) {

    x = variables[1]
    r = variables[2]

    psi = sqrt(x*(1 - x))*exp((-1/2)*(((kappa**2)*(r**2)*x*(1 - x)) +
    ((m**2)/((kappa**2)*x*(1 - x)))))

    psi_dr2 = (kappa**2)*((x*(1 - x))**(3/2))*((kappa**2)*(r**2)*x*(1 - x) - 2)*
    exp((-1/2)*(((kappa**2)*(r**2)*x*(1 - x)) + ((m**2)/((kappa**2)*x*(1 - x)))))

    (1/2)*pi*r*((psi**2) + (2*(m**2)*(psi**2) - 2*psi*psi_dr2)/(x*(1 - x)*(M**2)) +
    ((m**4)*(psi**2) + (psi_dr2**2) - (2*(m**2)*psi*psi_dr2))/((x*(1 - x)*(M**2))**2))

}

N_L = 1/sqrt(adaptIntegrate(Normalise_L, lowerLimit = c(0, 0),
                                         upperLimit = c(1, 25))$integral)
```

R Program

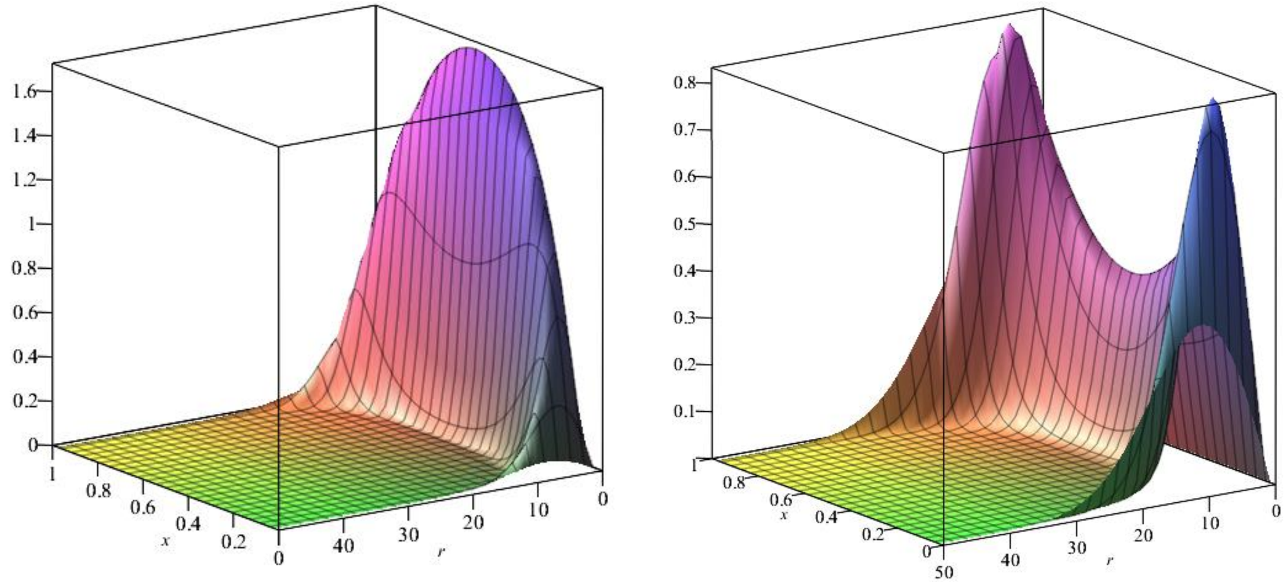
- `adaptIntegrate()` cannot integrate to infinity.





R Program

- Upper limits on normalisation integrals were set to 25 (longitudinal) and 100 (transverse).



Optimisation – Simplification

- Most functions were simplified by hand.
- For example, the b-CGC scattering amplitude:

$$\mathcal{I}\text{m}\mathcal{A}_\lambda(s,t;Q^2) = \sum_{h,\bar{h}} \int \mathrm{d}^2\underline{r}\mathrm{d}x \underbrace{\Psi^{\gamma^*,\lambda}_{h,\bar{h}}(x,r;Q^2)}_{\substack{\text{Light-front}\\ \text{perturbation theory}}} \underbrace{\Psi^{V,\lambda}_{h,\bar{h}}(x,r)^*}_{\substack{\text{Light-front}\\ \text{holography}}} e^{-ix\underline{r}\cdot\underline{\Delta}} \underbrace{\mathcal{N}(x,r,\Delta)}_{\text{QCD}}$$

$$\Psi^{V,L}_{h,\bar{h}}(x,r) = \frac{1}{2}\delta_{h,\bar{h}}\left(1 + \frac{m_f^2 - \nabla_r^2}{x(1-x)M_V^2}\right)\Psi_L(x,r)$$

```
2 *

(sqrt(N_c/(pi))*q_e*e*x*(1 - x)*sqrt(Q2)*K(epsilon*r))/pi *

(N_L/(2*sqrt(2)))*((1 + ((m**2)/(x*(1 - x)*(M**2))))
*(psi) - (1/(x*(1 - x)*(M**2)))*(psi_dr2)) *

((2*pi)**2)*r*b*N *

J(x*r*sqrt(t_in))*J(b*sqrt(t_in))
```

Optimisation – Vectorisation

- Everything in R is a vector.
- Programs written to take advantage of this are faster.
- All functions were vectorised manually, with special R functions, or with the `Vectorize()` function.

$$\mathcal{N}(x,r) = \begin{cases} \mathcal{N}_0 \left(\frac{rQ_s}{2}\right)^{2\left(\gamma_s + \frac{1}{\kappa\lambda \ln\left(\frac{1}{x}\right)} \ln\left(\frac{2}{rQ_s}\right)\right)} & rQ_s \leq 2 \\ 1 - e^{-A\ln^2(BrQ_s)} & rQ_s > 2 \end{cases}$$

```
N = ifelse((r*Q_s/2) - 1 > 0,
           2 - 2*exp(-A*(log(B*r*Q_s))**2),
           2*N_nought*(r*Q_s/2)**(2*(gamma_s
           + (log(2/(r*Q_s)))/(9.9*lambda*log(1/x_Bj)))))
```

Optimisation – Tolerance

- The tolerance was increased on all integrals. (10x)
- The default is `1e-5`, but it was increased to `1e-2` without significantly affecting the precision of the calculations.

```
adaptIntegrate(A_L, lowerLimit = c(1e-10, 1e-10, 1e-10),
                    upperLimit = c(50, 0.5, 100),
                    0, 0.5, 50)$integral}

adaptIntegrate(A_L, lowerLimit = c(1e-10, 1e-10, 1e-10),
                    upperLimit = c(50, 0.5, 100),
                    0, 0.5, 50, tol = 1e-2)$integral}

[1] 0.0005195478
[1] 0.0005192687
```

Optimisation – Other Changes

- Comparisons against 0 are faster. Therefore, the conditions on the piecewise $N(x, r)$ and $N(x, r, b)$ functions were rewritten. (2.7x)
- Because all functions are symmetric in x, the upper limit was set to 0.5, then the result was multiplied by 2. (1.8x)
- Using a faster processor

R Program

- The `VMP()` function can output `d_sigma`, `sigma`, and `ratio` values, as well as errors.
- The output is an R data frame.
- The data from the data frame can be easily displayed or called by another function later.

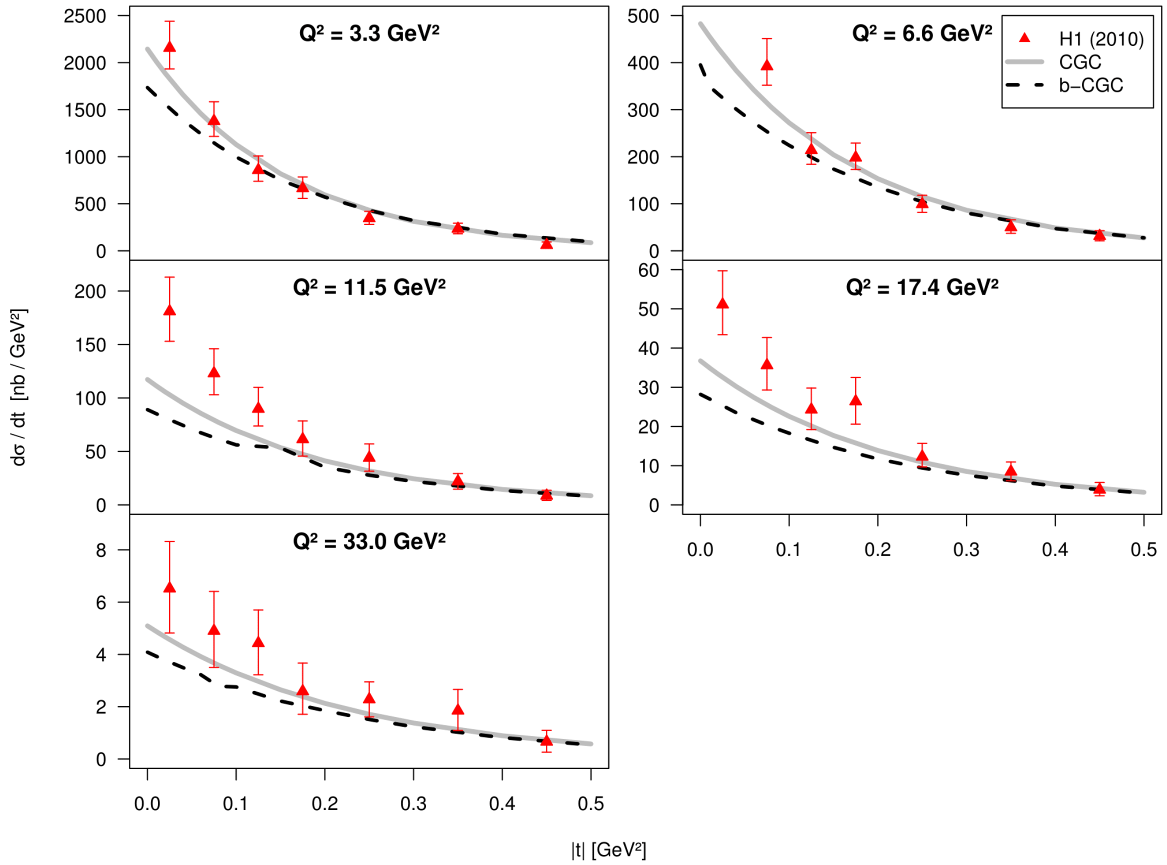
```
VMP(model = "bCGC", parameters = "Amir", meson = "rho", W = 75, Q2 = bCGC_Q2,
    return = "sigma")$sigma
```

W <dbl>	Q2 <dbl>	t <dbl>	d_sigma <dbl>	sigma <dbl>	sigma_error <dbl>	ratio <dbl>	ratio_error <dbl>
75	1	0	1.649950e+04	2.509534e+03	4.932490024	0.5821071	0.003149744
75	2	0	5.748792e+03	9.267990e+02	3.437952933	1.0059009	0.007491377
75	15	0	6.571915e+01	1.369358e+01	0.028118749	4.2283055	0.039163778
75	20	0	2.998957e+01	6.484682e+00	0.053389937	5.2671936	0.061027244
75	25	0	1.613772e+01	3.505803e+00	0.028034274	6.0218509	0.079882200
75	30	0	9.592851e+00	2.158338e+00	0.006816969	7.1404210	0.025930675
75	40	0	3.905281e+00	9.612168e-01	0.009404427	9.1595874	0.832325682
75	50	0	2.065010e+00	5.040970e-01	0.007091700	10.7000782	1.532241885
75	75	0	6.077957e-01	1.479488e-01	0.008538513	14.4947252	11.600263645
75	100	0	2.468990e-01	6.423527e-02	0.007816554	18.9680613	8.857521258

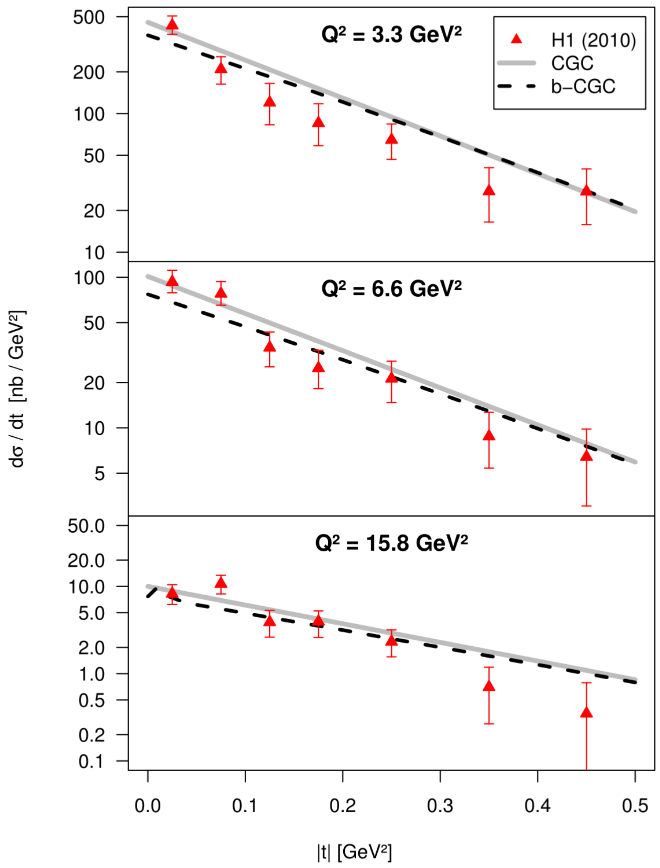
Results

- Plot the differential cross section, cross section, and ratio for ρ and φ production using both the CGC and b-CGC models.
  - Differential cross section vs |t|
  - Cross section vs Q^2
  - Cross section vs W
  - Ratio vs Q^2
- Compare to H1 and ZEUS data.

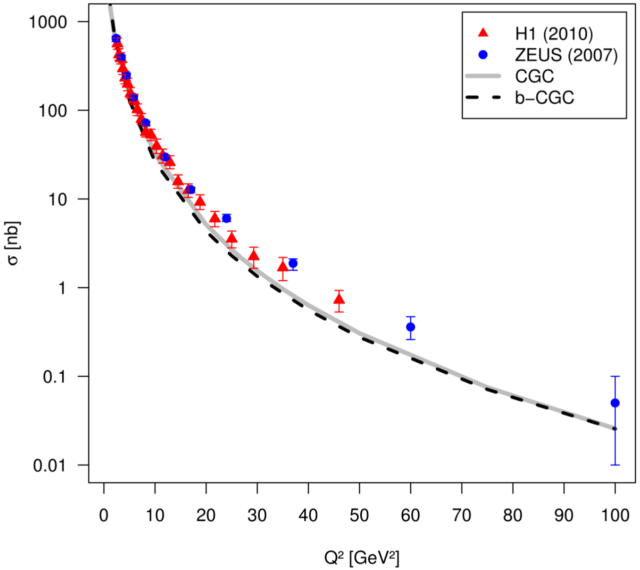
dσ/dt vs |t| (ρ)

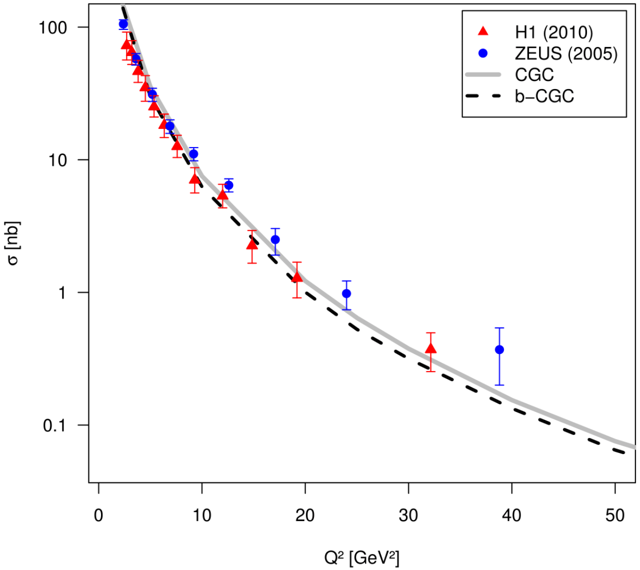

Q² = 3.3 GeV²
Q² = 6.6 GeV²
Q² = 11.5 GeV²
Q² = 17.4 GeV²
Q² = 33.0 GeV²
H1 (2010)
CGC
b–CGC
dσ / dt [nb / GeV²]
|t| [GeV²]
2500
2000
1500
1000
500
0
400
300
200
100
150
50
60
40
30
20
10
8
6
4
2
0.0
0.1
0.2
0.3
0.4
0.5

dσ/dt vs |t| (φ)

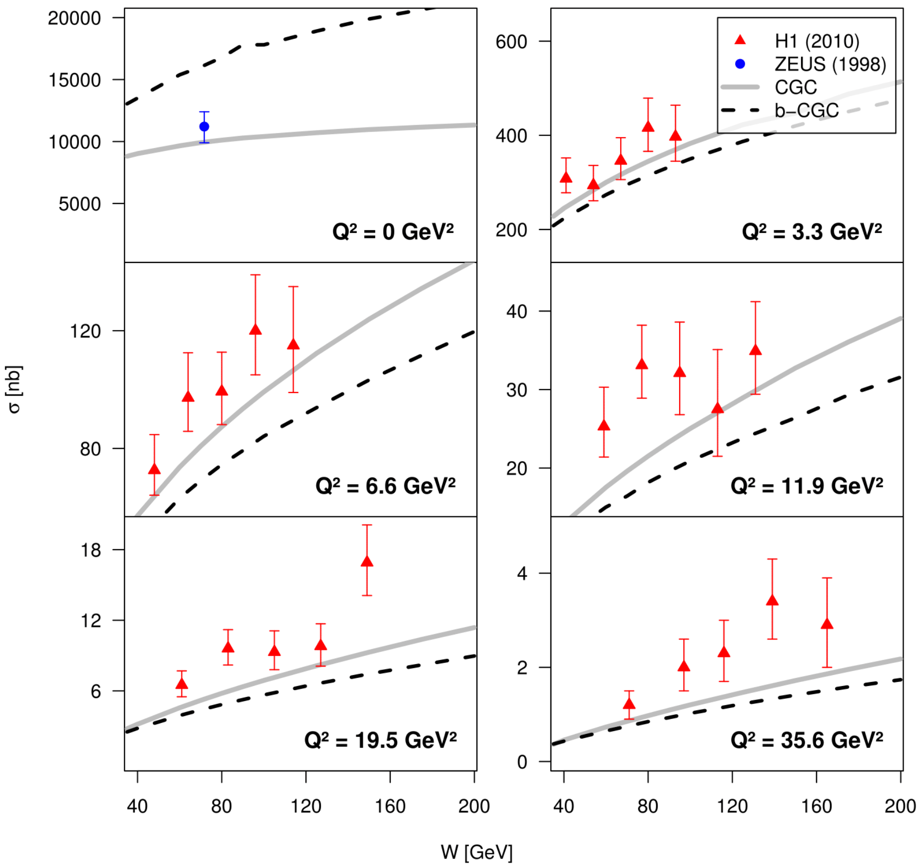

Q² = 3.3 GeV²
H1 (2010)
CGC
b–CGC
Q² = 6.6 GeV²
Q² = 15.8 GeV²
dσ / dt [nb / GeV²]
|t| [GeV²]
500
200
100
50
20
10
100
50
20
10
5
50.0
20.0
10.0
5.0
2.0
1.0
0.5
0.2
0.1
0.0
0.1
0.2
0.3
0.4
0.5

σ vs Q2 (ρ and φ)

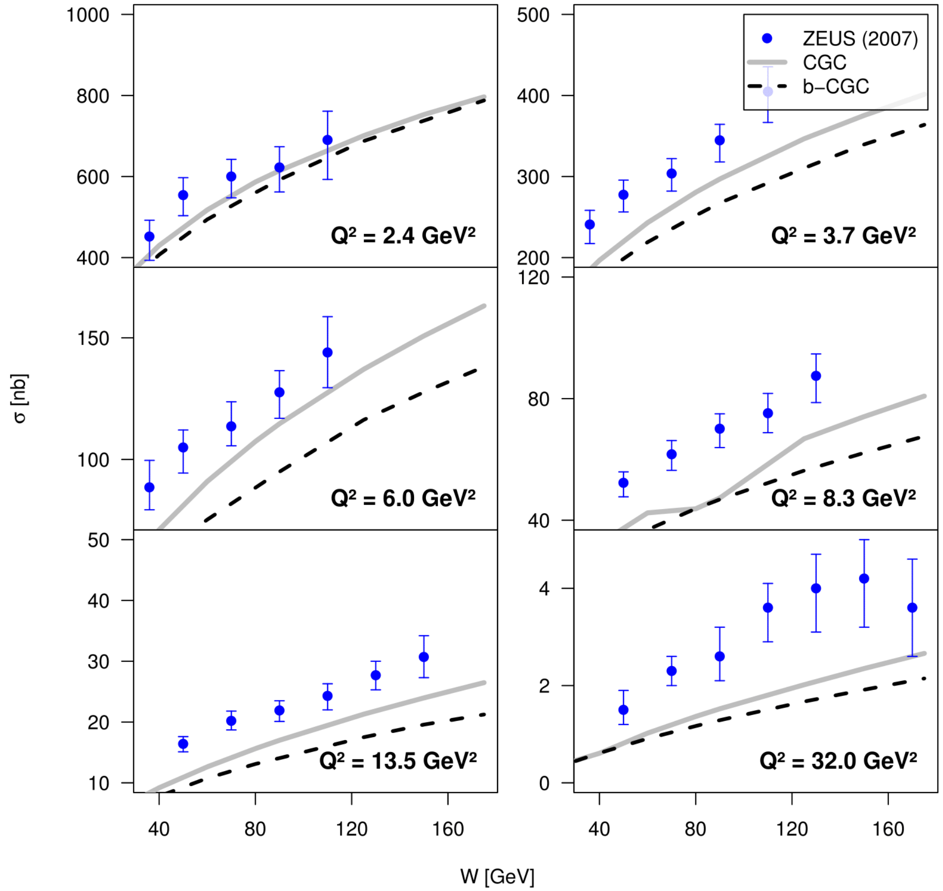

H1 (2010)
ZEUS (2007)
CGC
b−CGC
σ [nb]
1000
100
10
1
0.1
0.01
0
10
20
30
40
50
60
70
80
90
100
Q² [GeV²]


H1 (2010)
ZEUS (2005)
CGC
b−CGC
σ [nb]
100
10
1
0.1
0
10
20
30
40
50
Q² [GeV²]

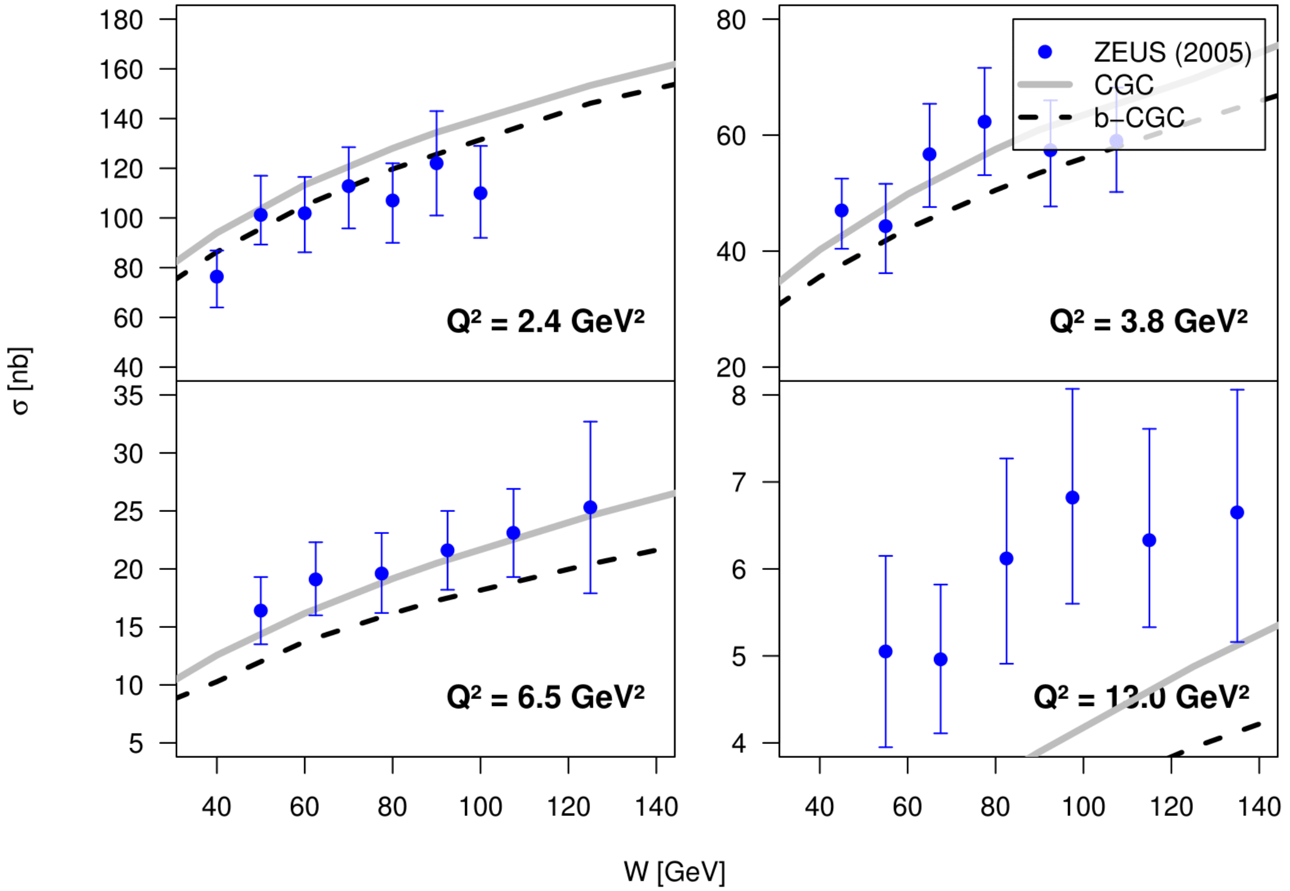
σ vs W (ρ, #1)


H1 (2010)
ZEUS (1998)
CGC
b–CGC
Q² = 0 GeV²
Q² = 3.3 GeV²
Q² = 6.6 GeV²
Q² = 11.9 GeV²
Q² = 19.5 GeV²
Q² = 35.6 GeV²
20000
15000
10000
5000
600
400
200
120
80
40
30
20
18
12
6
4
2
0
40
80
120
160
200
σ [nb]
W [GeV]

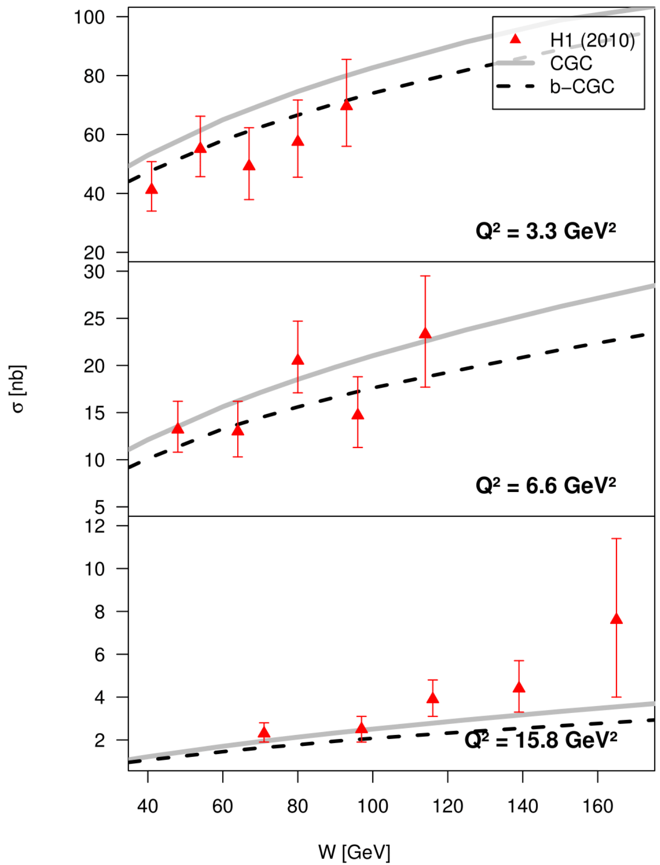
σ vs W (ρ, #2)


ZEUS (2007)
CGC
b−CGC
Q² = 2.4 GeV²
Q² = 3.7 GeV²
Q² = 6.0 GeV²
Q² = 8.3 GeV²
Q² = 13.5 GeV²
Q² = 32.0 GeV²
σ [nb]
W [GeV]
1000
800
600
400
500
300
200
120
150
100
50
80
40
30
20
10
4
2
0
40
80
120
160

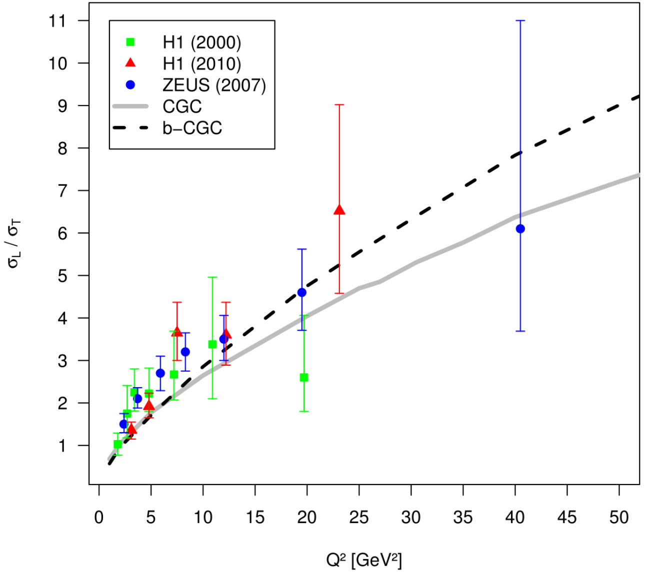
σ vs W (φ, #1)

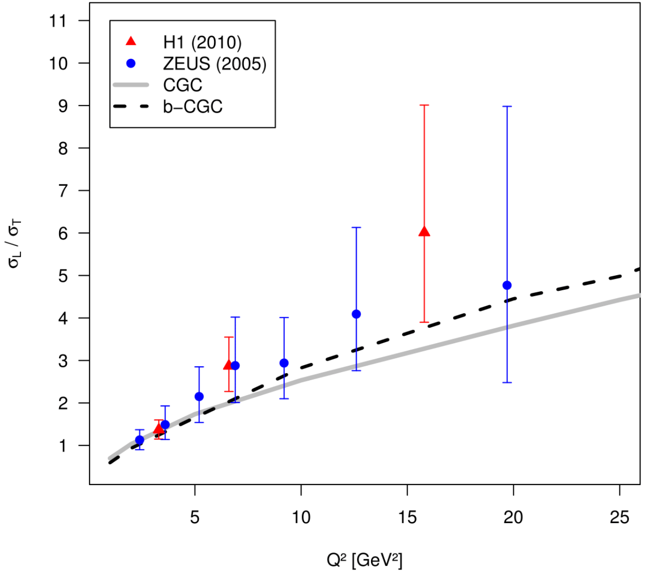

σ [nb]
W [GeV]
ZEUS (2005)
CGC
b–CGC
Q² = 2.4 GeV²
Q² = 3.8 GeV²
Q² = 6.5 GeV²
180
160
140
120
100
80
60
40
35
30
25
20
15
10
5
8
7
6
5
4
40
60
80
100
120
140

σ vs W (φ, #2)


H1 (2010)
CGC
b–CGC
Q² = 3.3 GeV²
Q² = 6.6 GeV²
Q² = 15.8 GeV²
σ [nb]
W [GeV]
100
80
60
40
20
30
25
20
15
10
5
12
10
8
6
4
2
40
60
80
100
120
140
160

R vs Q^2 (ρ and φ)


H1 (2000)
H1 (2010)
ZEUS (2007)
CGC
b–CGC
σL / σT
Q² [GeV²]

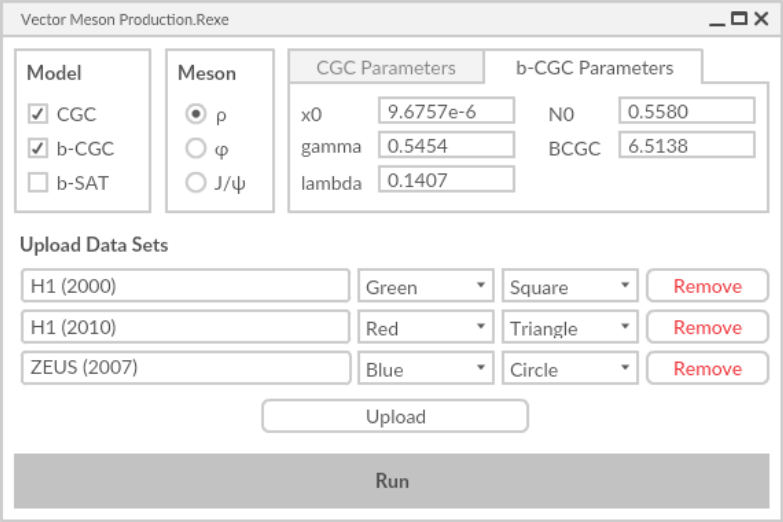

H1 (2010)
ZEUS (2005)
CGC
b–CGC
σL / σT
Q² [GeV²]

Conclusion

- In general, results are encouraging.
- A cross-check is needed to verify the numerical results.
- The light-front holographic wavefunction is a good description for the production of φ mesons.

Conclusion

- Future work:
  - Find and fix any bugs that may exist in the program
  - Expand the program to other mesons and models
  - Create a GUI to wrap the calculations together



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