UNIVERSITY OF ALBERTA

MUTE: A Program for High-Precision Calculations of Underground and Underwater Muon Intensities

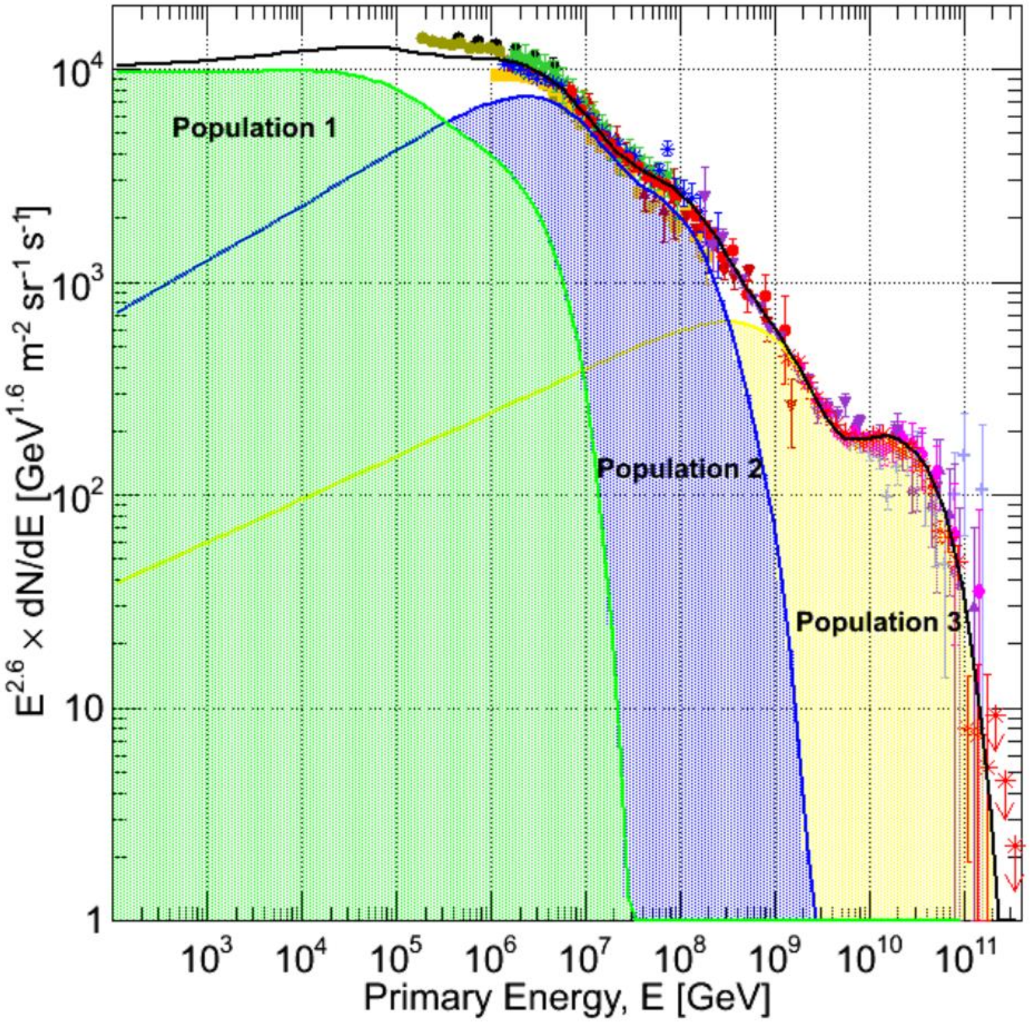
Ph.D. Defense

William Woodley

Supervisor: Prof. Marie-Cécile Piro

Cosmic Rays

- Atmospheric muons originate from cosmic rays (CRs).
- CRs are charged particles entering Earth's atmosphere from outer space.
- First discovered by Victor Hess in 1912, but many questions still remain today, particularly about their origin and composition.
- CRs can interact in the atmosphere to induce extensive air showers, which can produce muons.
- These muons can travel deep underground.
- Underground muons can be used to study the CR spectrum because their properties point back to the properties of the CRs they originate from.



[1] T. K. Gaisser, et al., *Cosmic Rays and Particle Physics* (2016).

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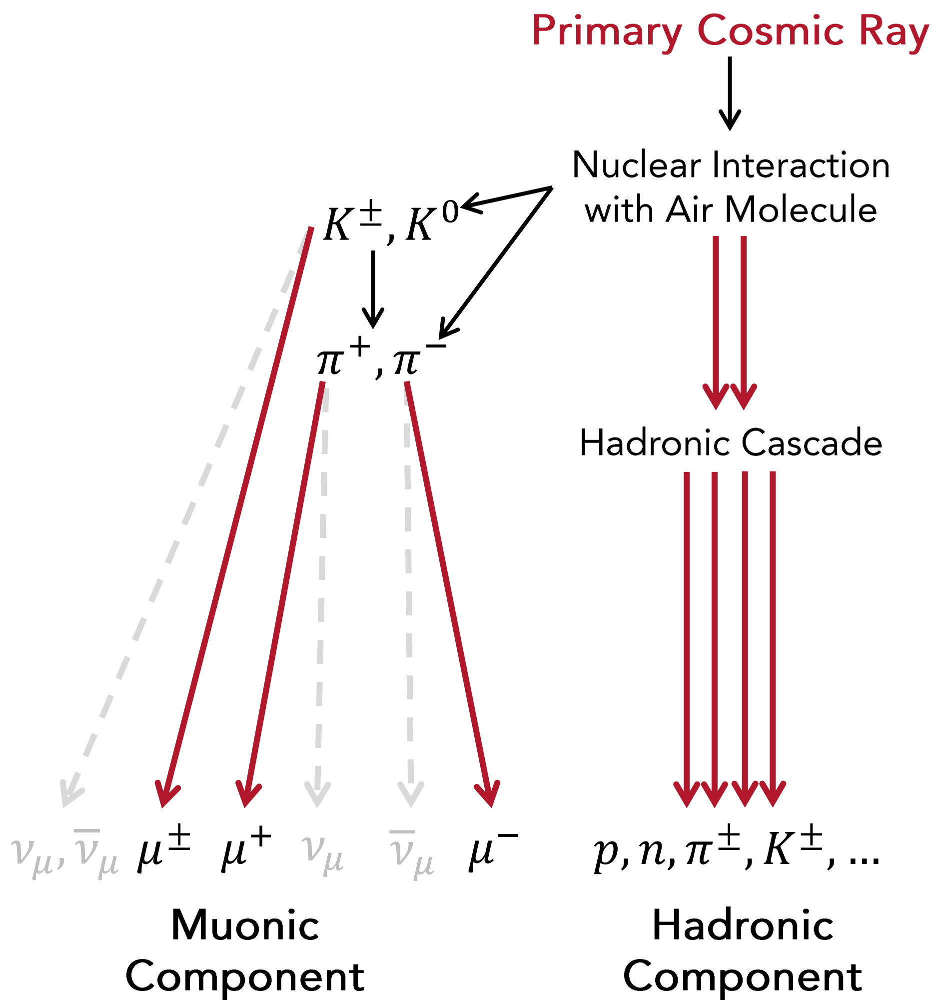
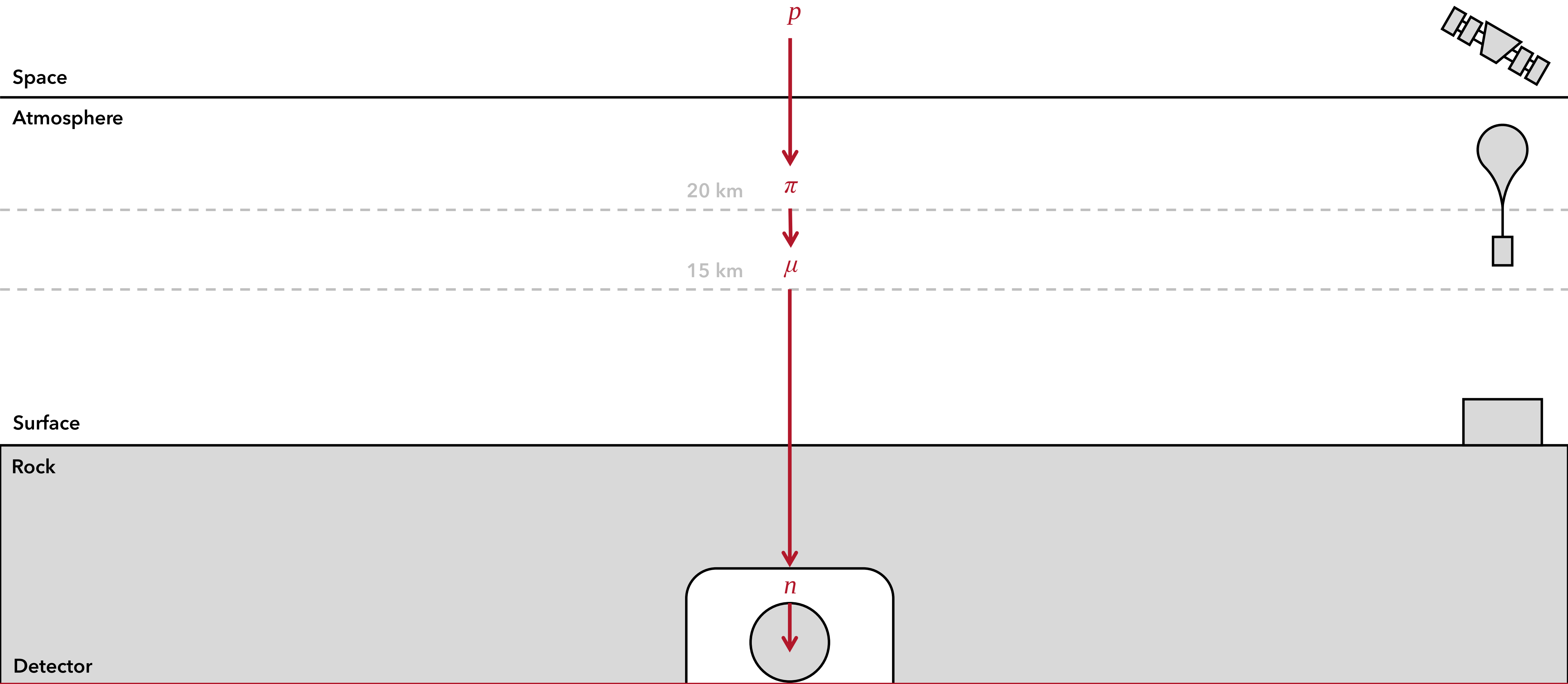


[2] A. Haungs et al. *J. Phys. Conf. Ser.* **632** (2015) 012011 [1504.06696].

Atmospheric Muons



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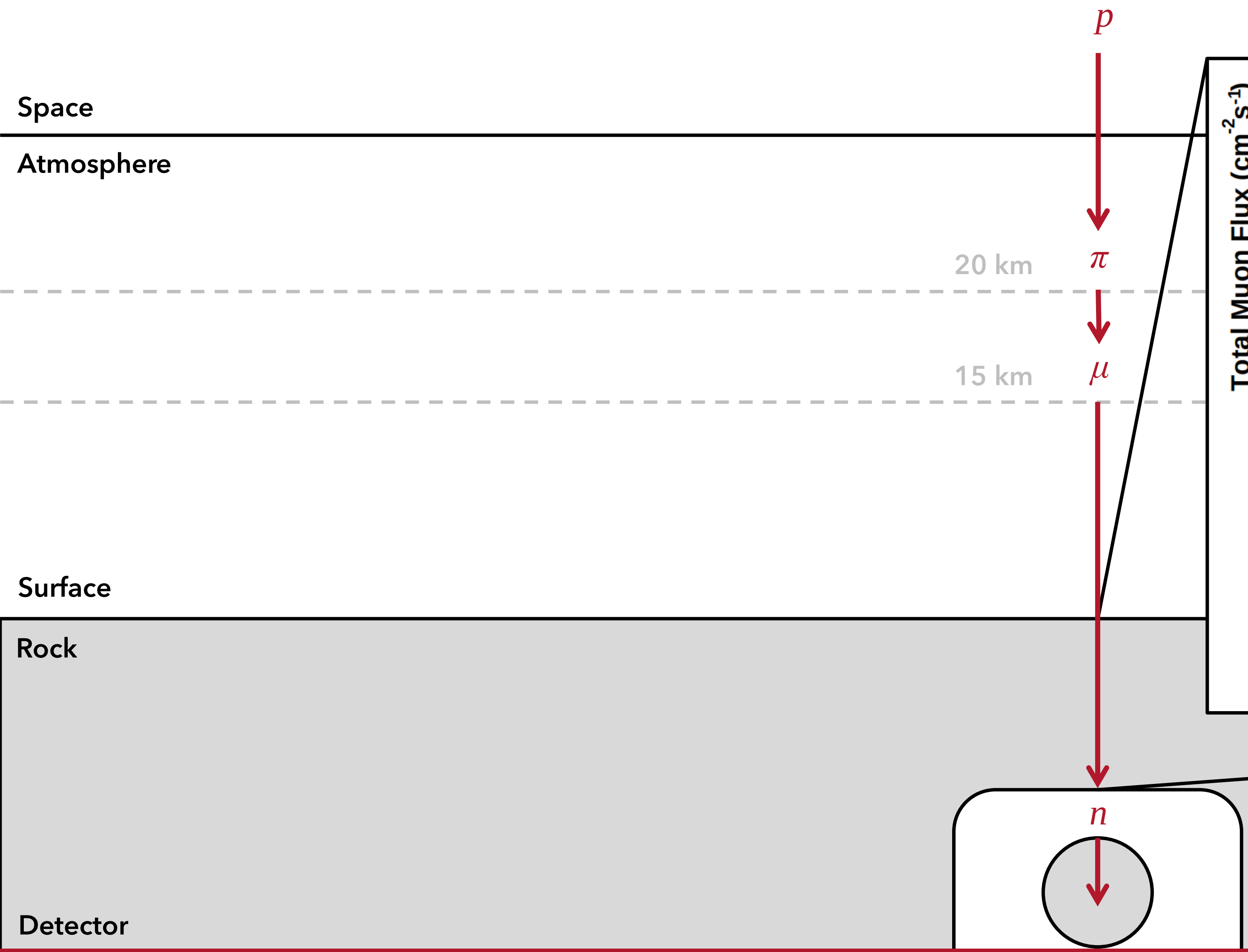
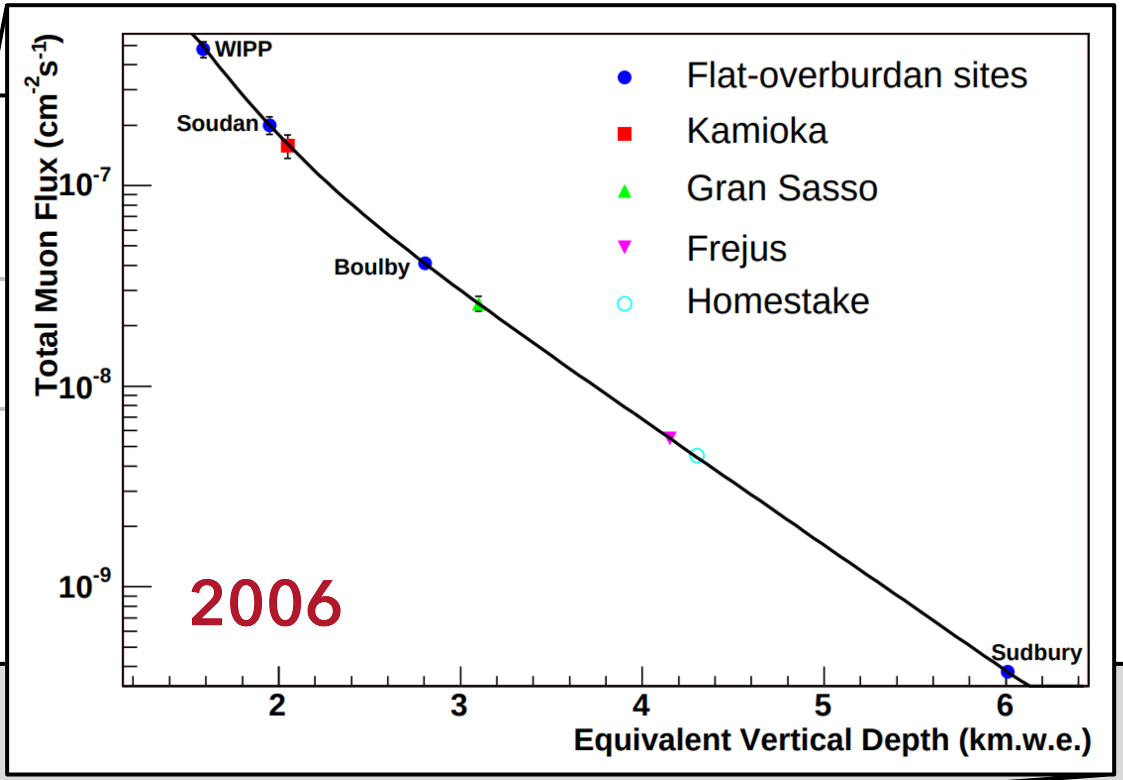
[3] D. Mei and A. Hime, *Phys. Rev. D* **73** (2006) 053004 [astro-ph/0512125].





Previous Methods

Analytical Calculations

- Calculations of muon intensities as semi-analytical solution of muon transport equations.
- Combine empirical formulas and Monte Carlo simulations.
- E. V. Bugaev, et al. (1998).

Disadvantages

- Lack rigorous treatment of errors.
- Calculations based on outdated data or assumptions.

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Parametric Fits

- Fit muon intensities to double-exponential formula:

$$I^{u}(h) = I_1 e^{-h/\lambda_1} + I_2 e^{-h/\lambda_2}$$

- Fit to vertical-equivalent muon intensity data.
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Muon Propagation Programs

- Various programs have been used over the past few decades.
- Notably MUM (2001) and MUSIC and MUSUN (2008).
- Based on analytical calculations or fits with Monte Carlo.

Disadvantages

- Not very accessible
- Computationally intensive
- Slow
- Limited input and output

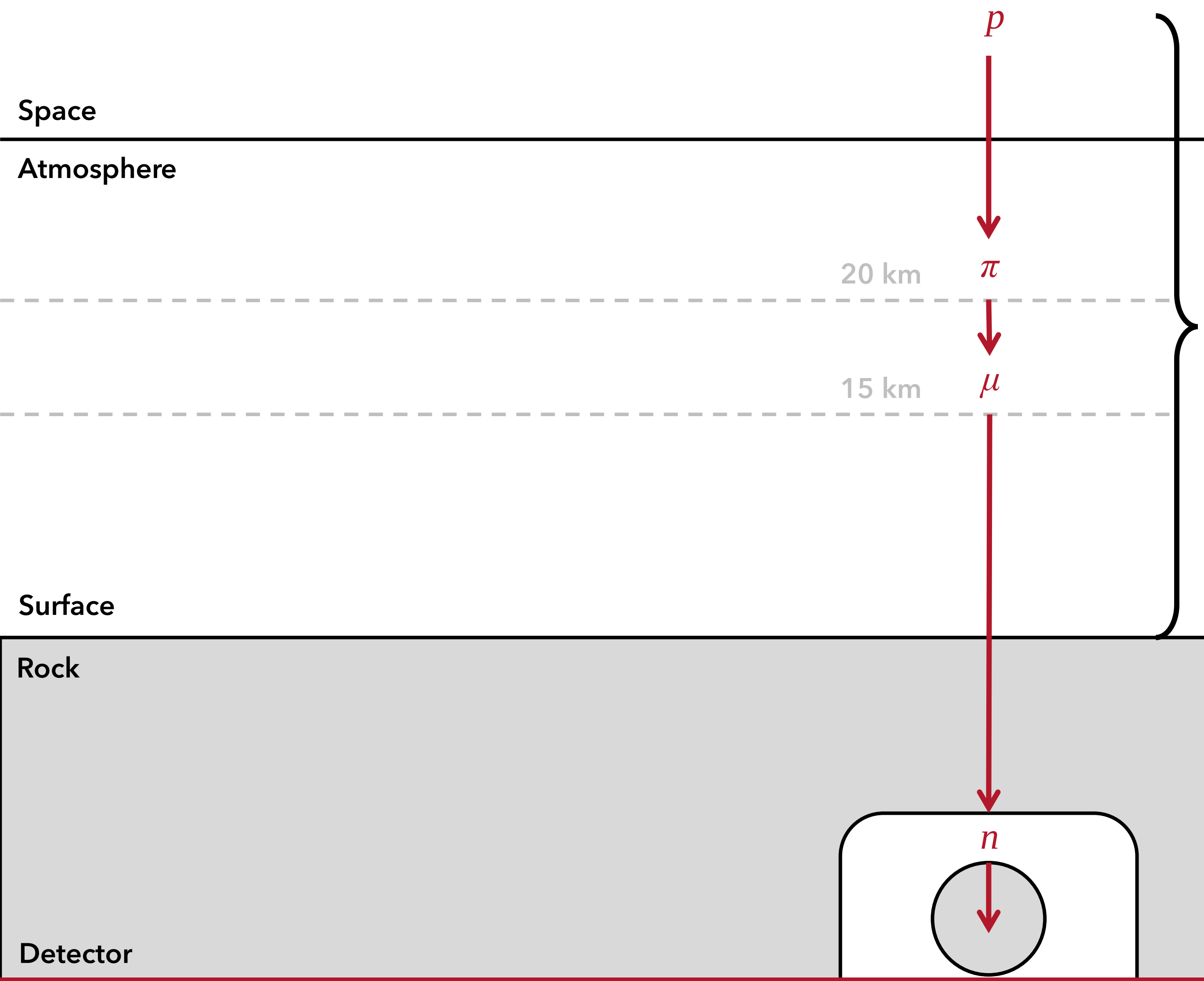
MUTE (MUon inTensity codE)

https://github.com/wjwoodley/mute

- Objectives:

1.	**Complete:**	Complete muon spectrum underground with physical units
2.	**Data-Independent:**	Forward prediction with no fits to already-existing underground data
3.	**Precise:**	Uncertainties smaller than previous programs
4.	**Flexible:**	Allow the user to set various models
5.	**Efficient:**	Provide results in a very short time (milliseconds)
6.	**Easy:**	Written in Python with a simple interface
7.	**Accessible:**	Open-source, BSD-3-Clause license, freely available on GitHub

Overview of Computational Method



MCEq

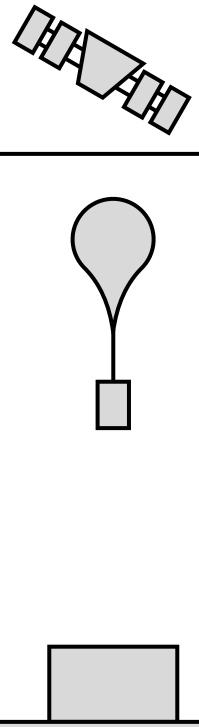
Matrix Cascade Equation

One-dimensional fast cascade equation solver

DAEMONFLUX

DAta-drivEn MuOn-calibrated atmospheric Neutrino FLUX

A combination of DDM and GSF fit to surface muon data



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[5] J. P. Yañez and A. Fedynitch, *Phys. Rev. D* **107** (2023) 123037 [2303.00022].
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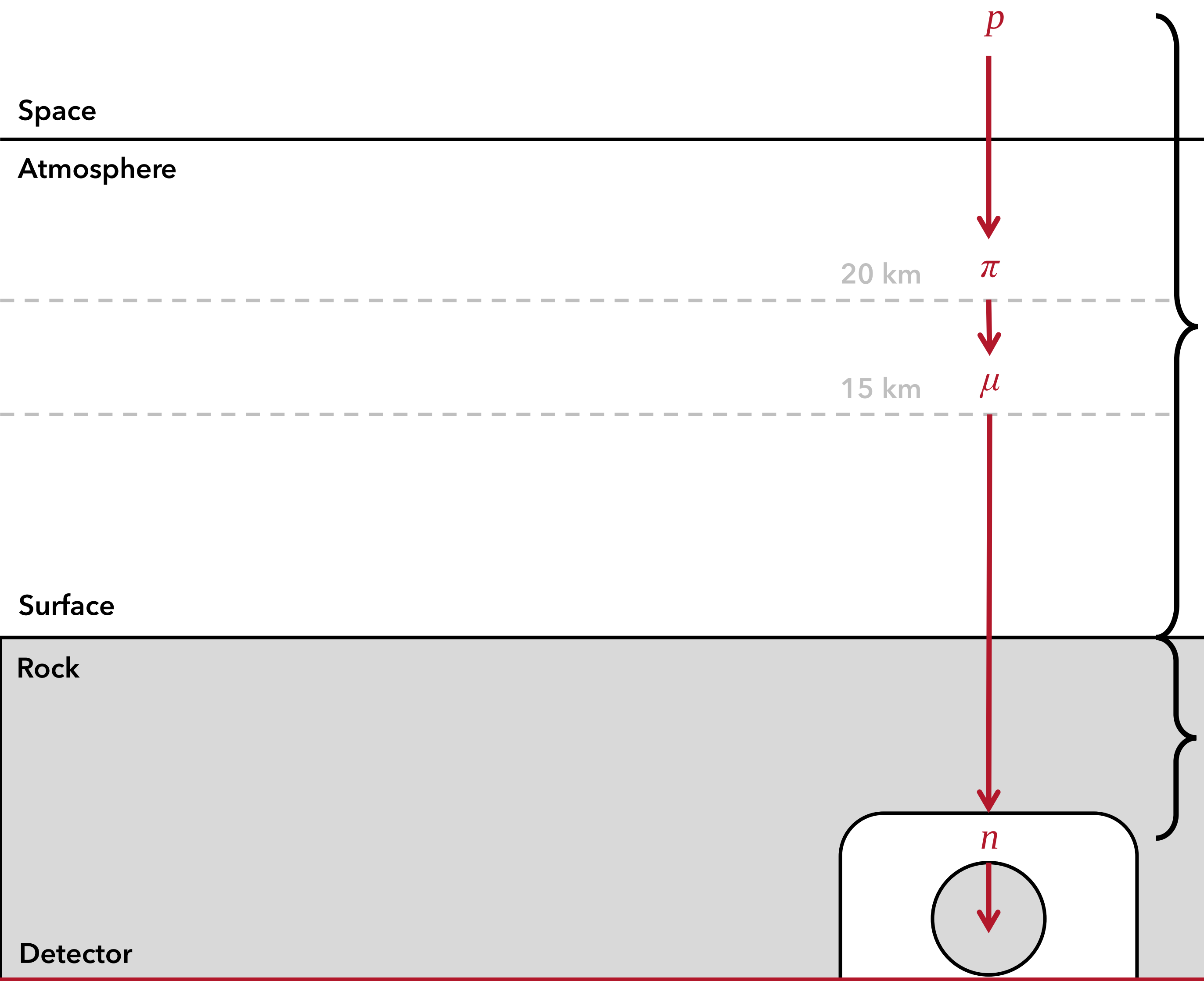

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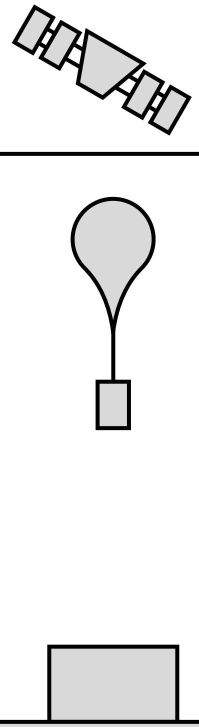

PROPOSAL

PRopagator with Optimal Precision and Optimised Speed for All Leptons

Monte Carlo code for lepton transport through long ranges of matter quickly

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Hadronic Interaction Models

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- Model the production of pions and kaons from CRs.

- **Event Generators:** Monte Carlo generators
- **Data-Driven Models:** Accelerator experiments

SIBYLL-2.3(c/d)

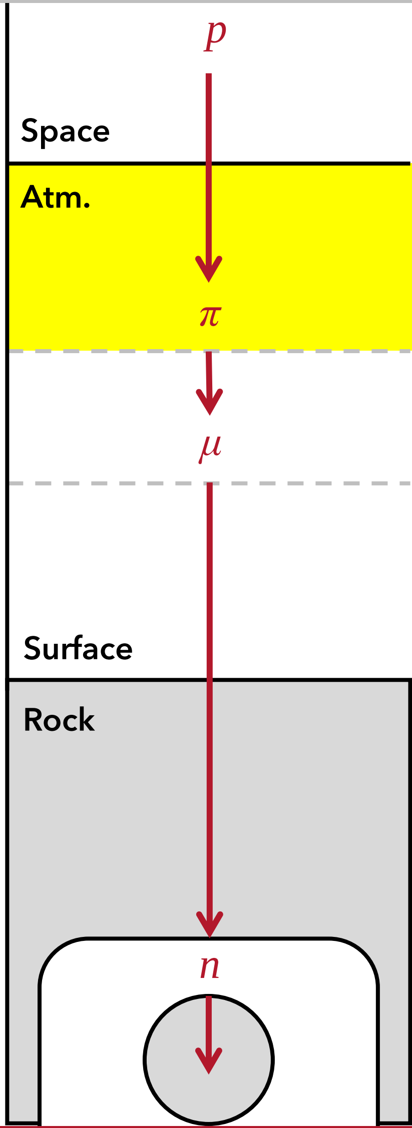
- Event generator for high-energy CRs
- Good for deep labs

DDM

- Based on CERN data
- Extrapolates to high-*E*
- Good for shallow labs

DAEMONFLUX

- Uses muon surface measurements to constrain DDM
- Achieves uncertainties less than 10% up to 1 TeV
- Most recent model with small uncertainties – model of choice



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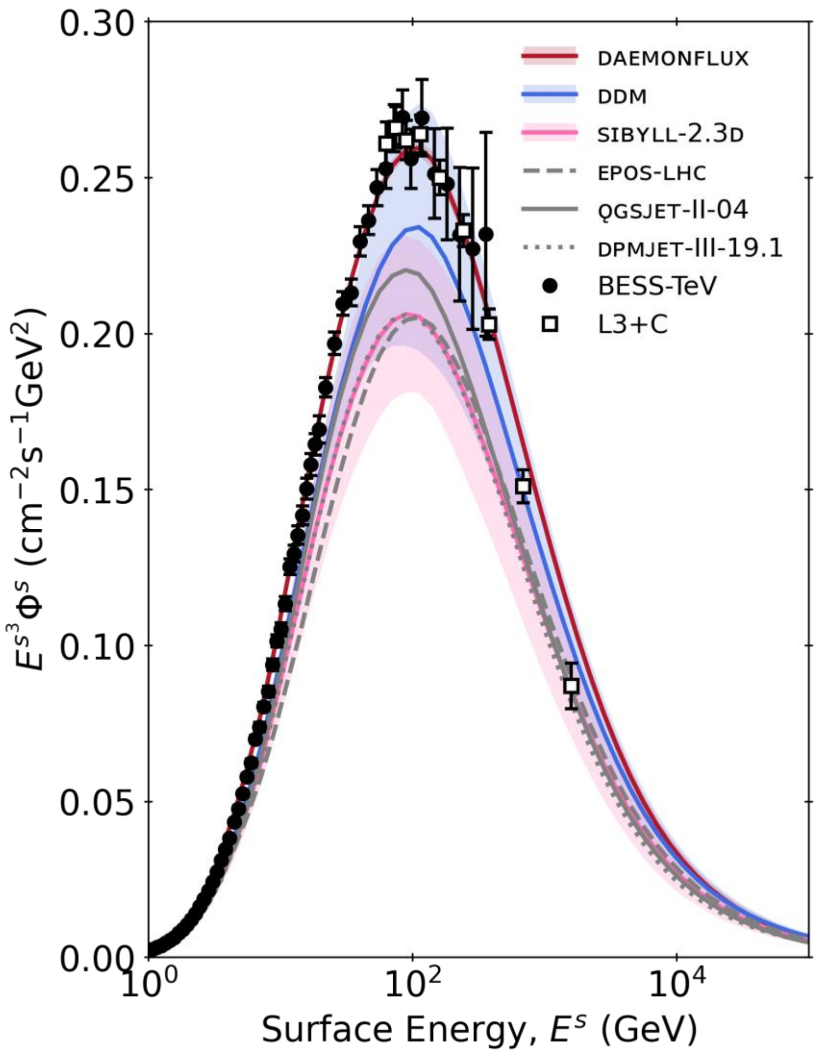
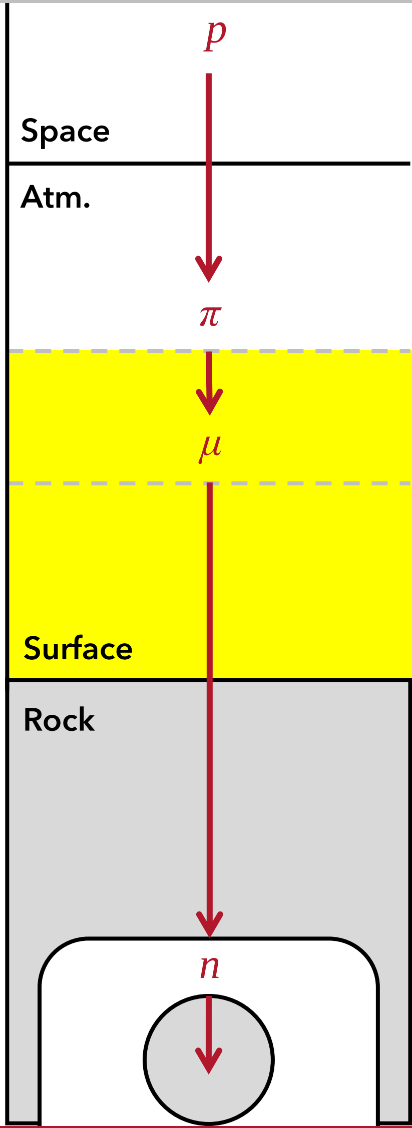
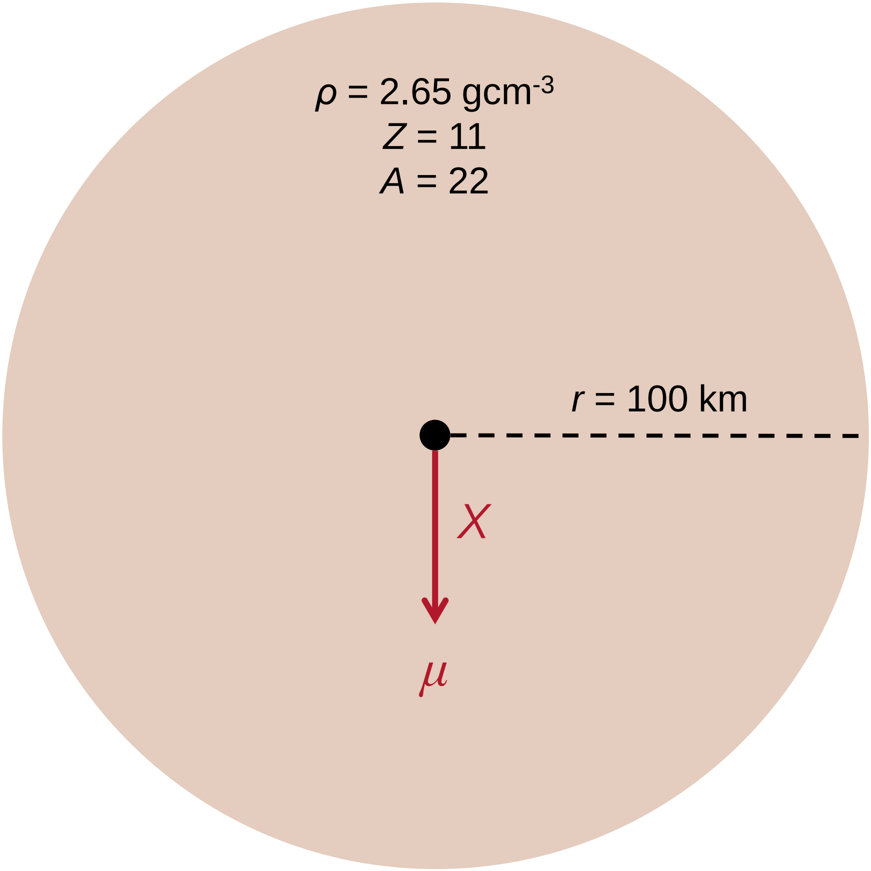
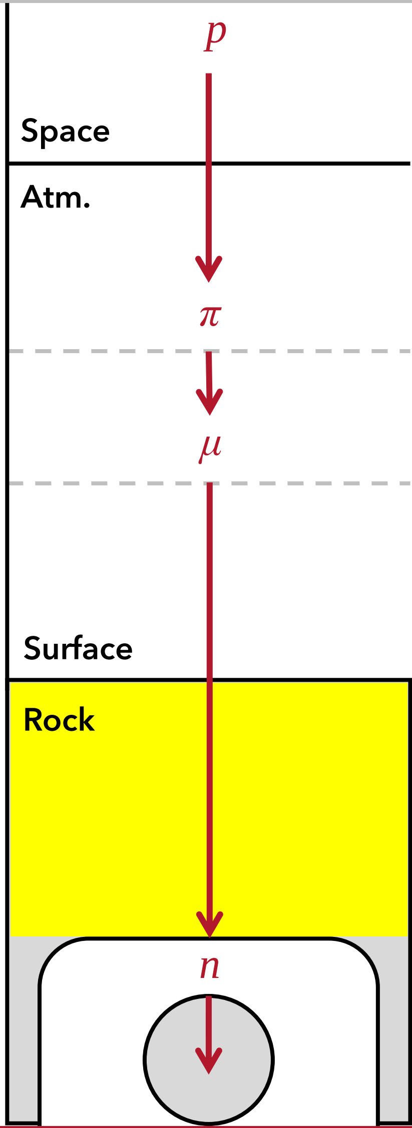




Propagation through Matter

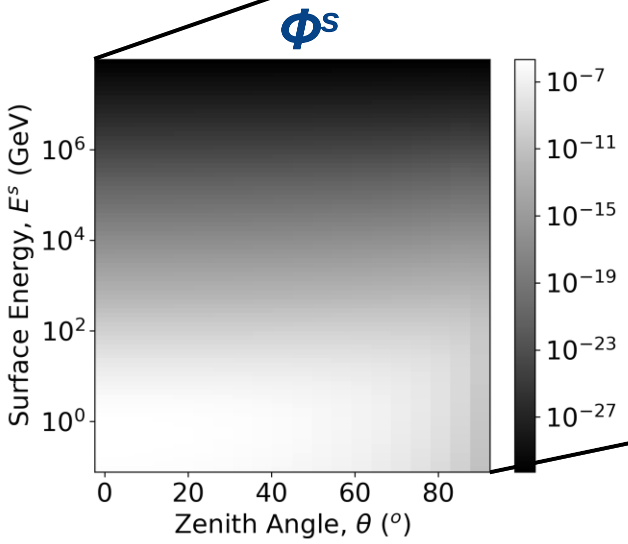
- Propagate muons through a sphere of rock. Loop over all surface energies and slant depths.
- Energy losses are calculated from parameterisation of cross-sections for:
  1. Ionisation
  2. Pair production
  3. Bremsstrahlung
  4. Photonuclear interactions
- Count how many muons survive and divide by the number of muons thrown (10^6) to calculate survival probabilities.
- Information stored in surface-to-underground transfer tensors. Transfer tensors take a week to compute on a computing cluster.
- Several pre-computed files are supplied with MUTE.



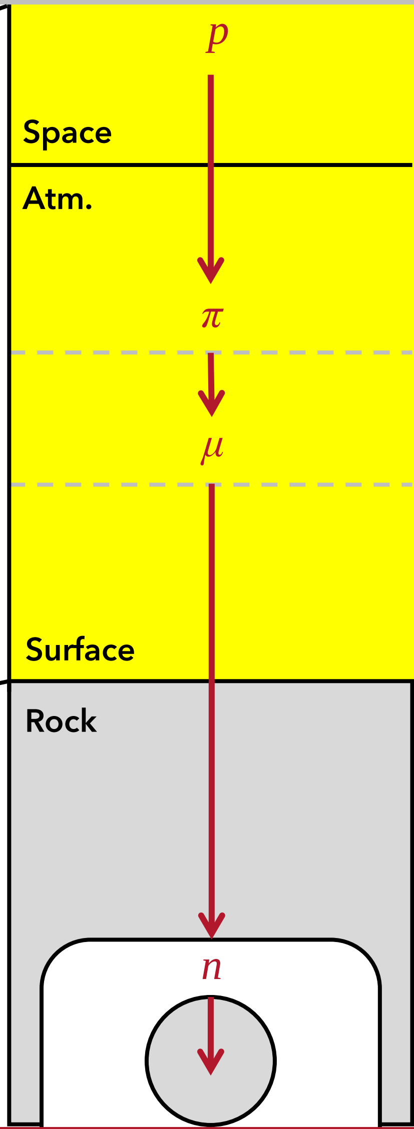


Overview of Computational Method

- A convolution is done between the surface fluxes and transfer tensor to calculate underground fluxes.



MCEq or DAEMONFLUX
Surface Fluxes



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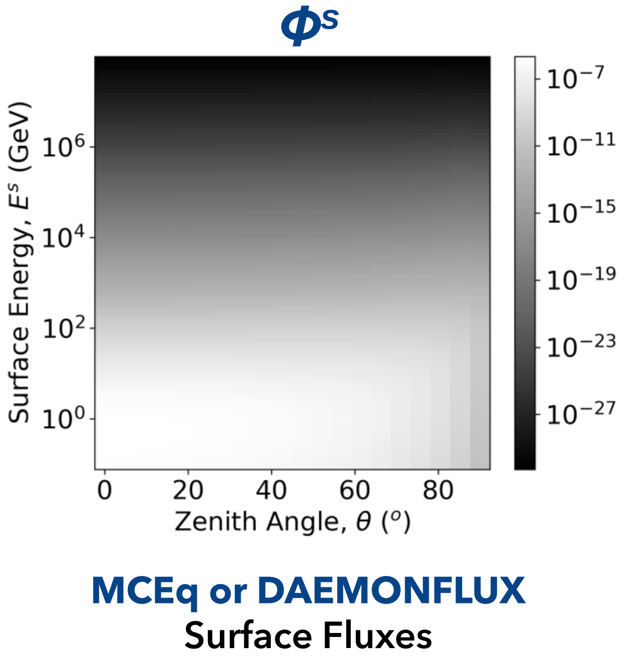

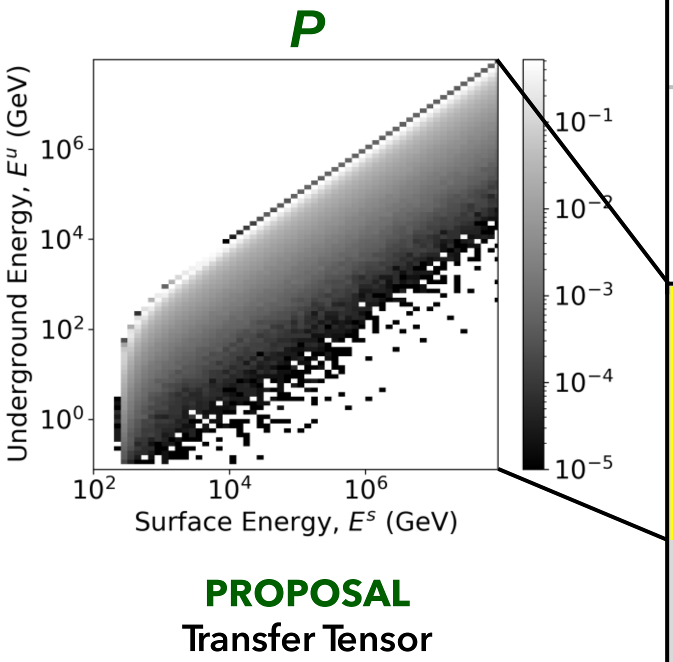

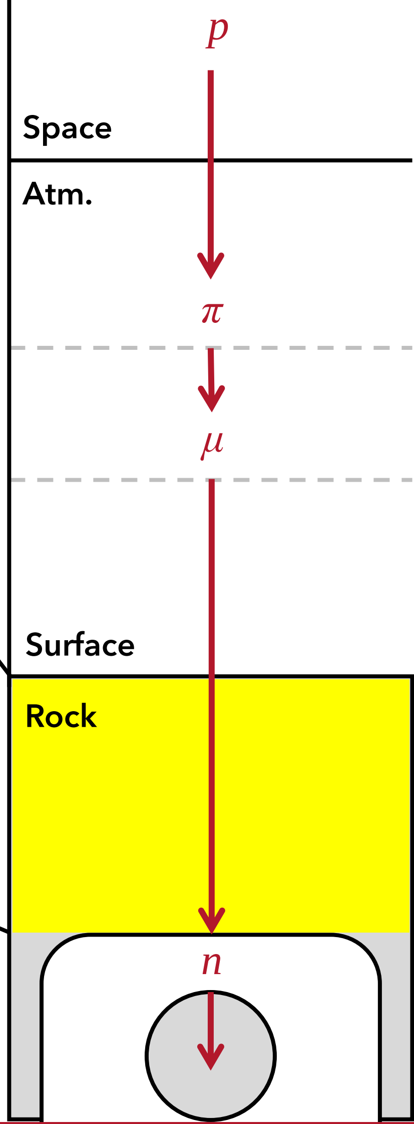

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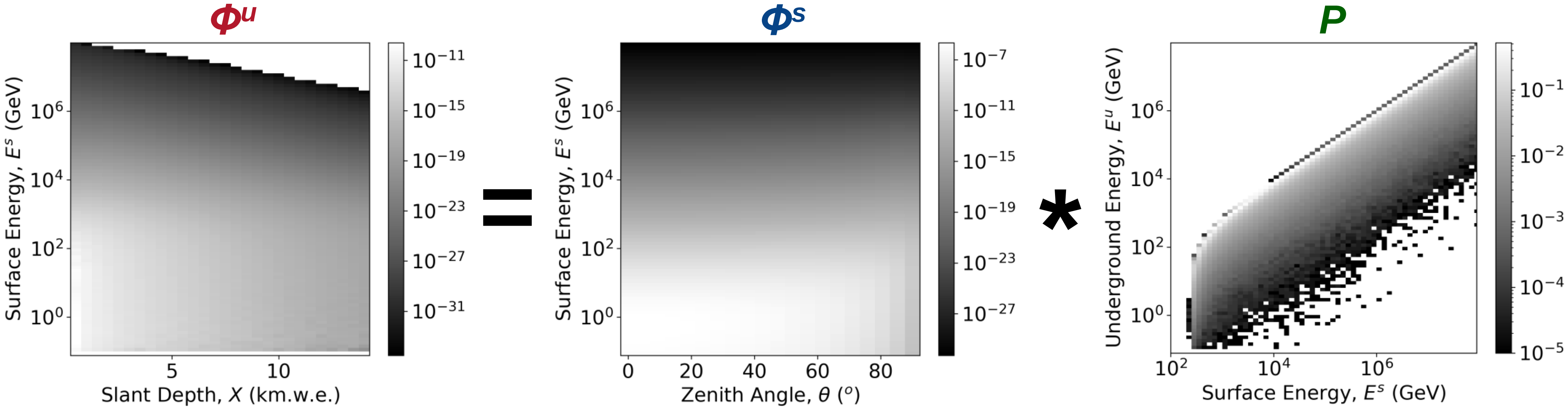
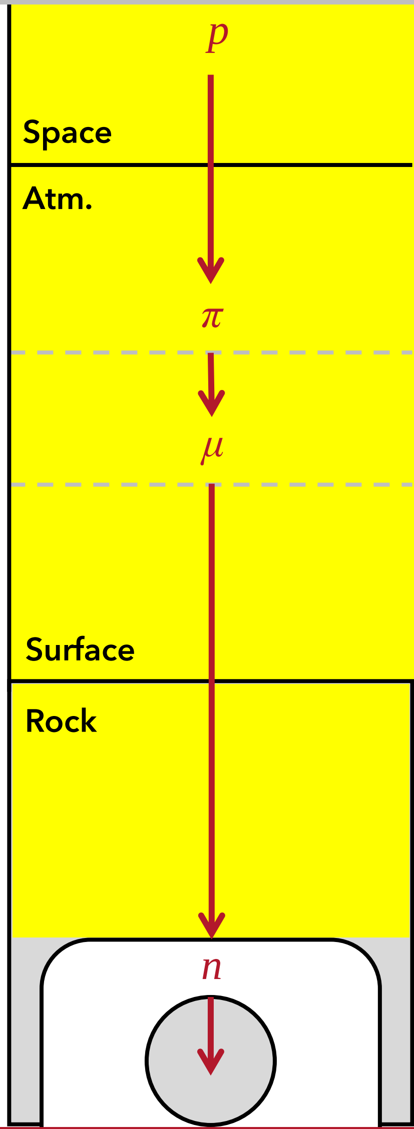
$$\Phi^u(E_j^u, X_k, \theta_k) = \sum_i \Phi^s(E_i^s, \theta_k) P(E_i^s, E_j^u, X_k) \left(\frac{\Delta E_i^s}{\Delta E_j^u}\right)$$



MUTE
Underground Fluxes

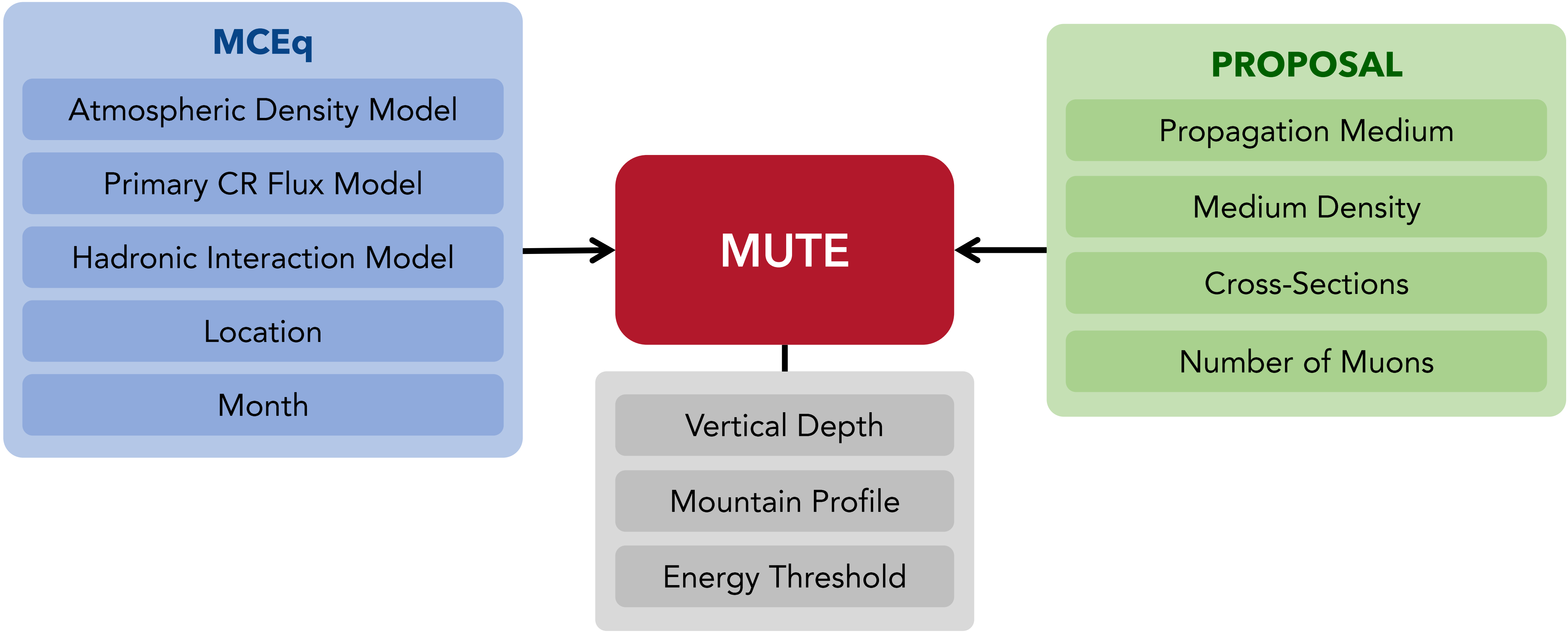
MCEq or DAEMONFLUX
Surface Fluxes

PROPOSAL
Transfer Tensor



Overview of Computational Method

- Written with a modular design so surface fluxes and transfer tensors can be changed out independently of each other.

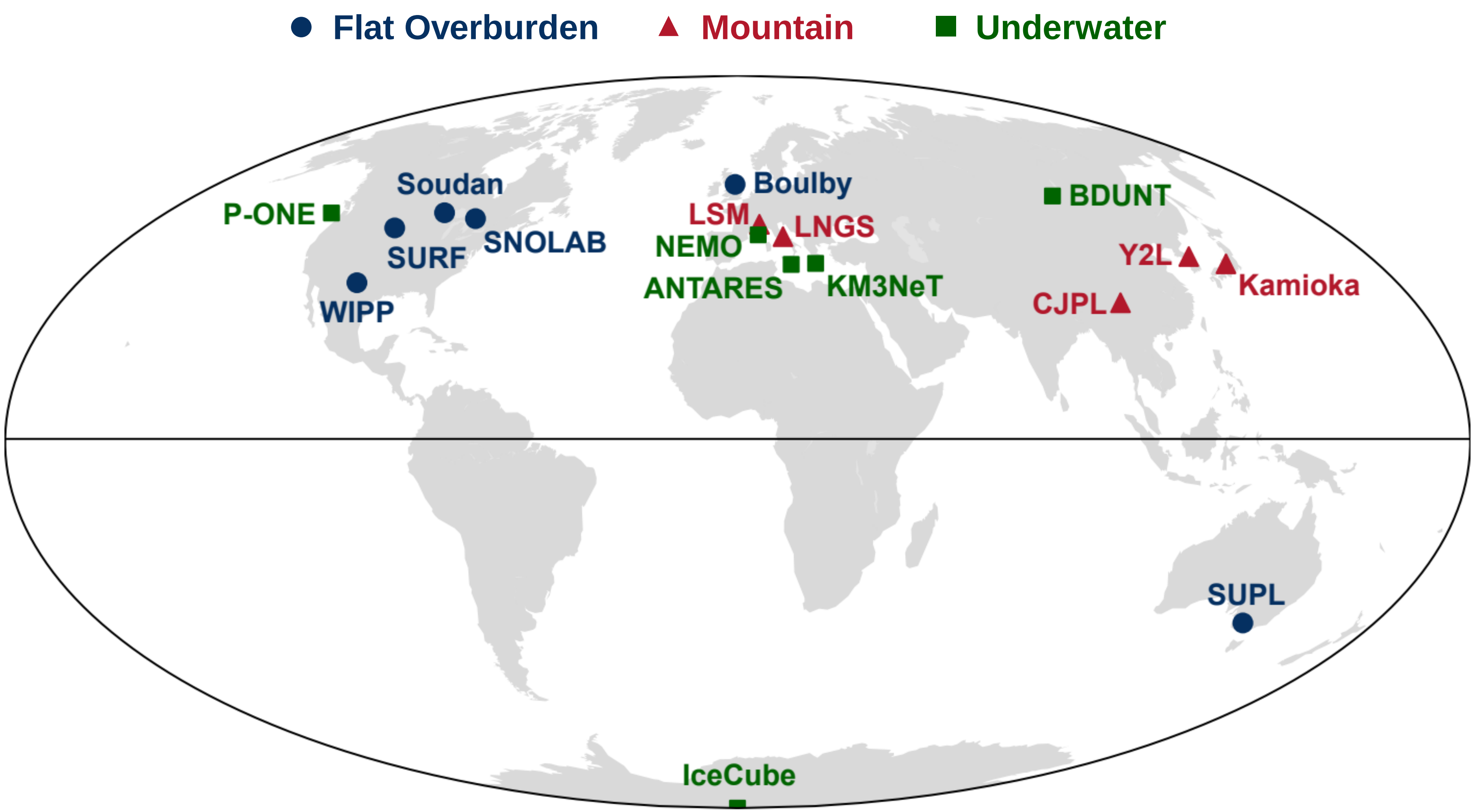


Method Comparison

Feature	MUTE	Calculations	Parametric Fits	Programs
Complete	✓	✓	✗	✓
Data-Independent	✓	✓	✗	✗
Precise	✓	✗	✗	✗
Flexible	✓	✗	✗	✓
Efficient	✓	✗	✓	✗
Easy	✓	✗	✓	✓
Accessible	✓	✓	✓	✗

What MUTE Offers

- Physical Observables:
  - Flux
  - Intensity
  - Angular Distribution
  - Energy Spectrum
  - Total Flux
- Environments:
  - Surface
  - Underground
  - Underwater



Underground Intensity

- Defined as underground flux integrated over underground energy of the muon:

$$I^u(X,\theta) = \int_{E_{\text{th}}}^{\infty} \Phi^u(E^u, X, \theta) \mathrm{d}E^u$$

- Vertical-equivalent intensity is defined as:

$$I^u_{eq}(X) = I^u(X,\theta)\cos(\theta)$$

- Compared to true vertical intensity:

$$I^u_{tr}(X) = I^u(X, \theta = 0)$$

- Experimental data over the decades has been published as vertical-equivalent intensity, which is a poor approximation for $\theta > 20°$.

Underground Intensity

- Excellent agreement between the prediction and data over the entire depth range.
- Data seems to follow the shape of Mei & Hime better.
- The downwards slant of the data in the ratio plot is because data is vertical-equivalent intensity whereas MUTE is true vertical intensity.
- Predictions match data well for water as well.
- Results were published in March 2022:

On the Accuracy of Underground Muon Intensity Calculations

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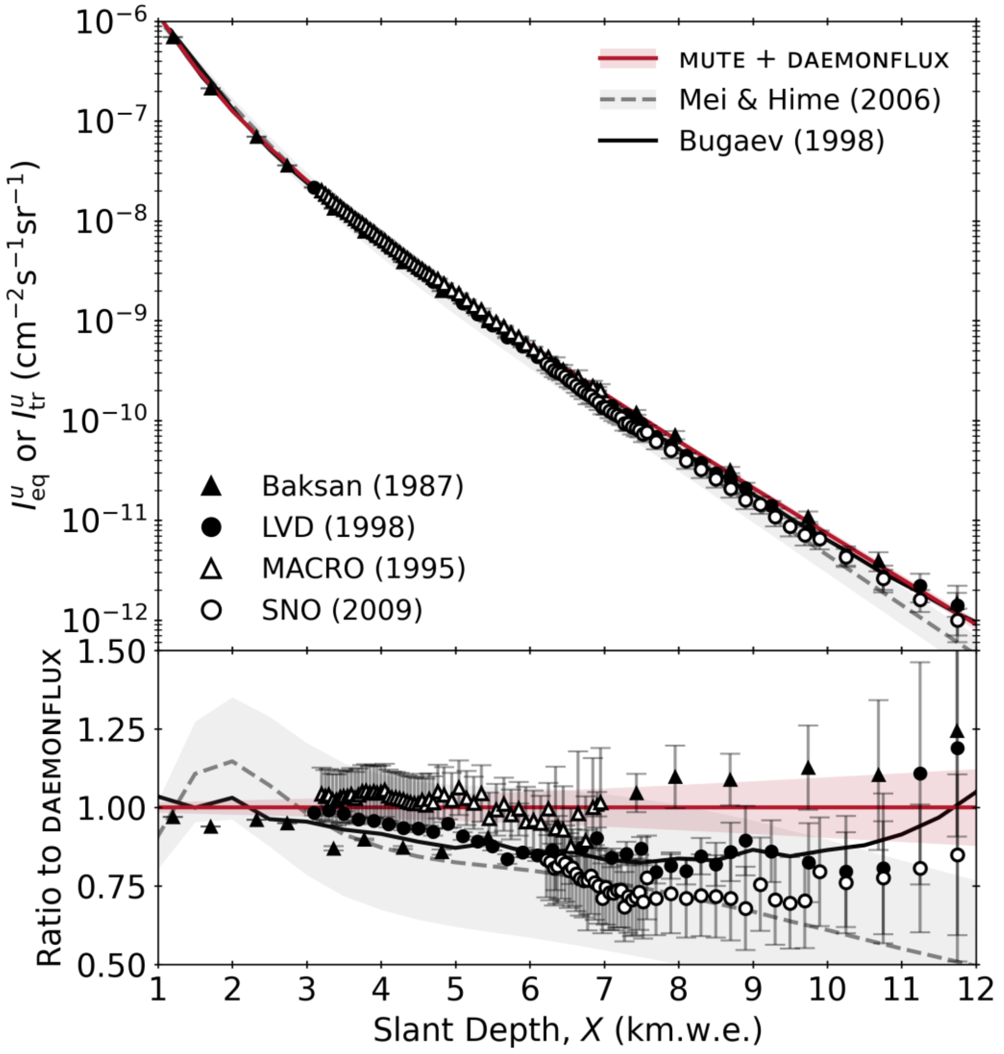

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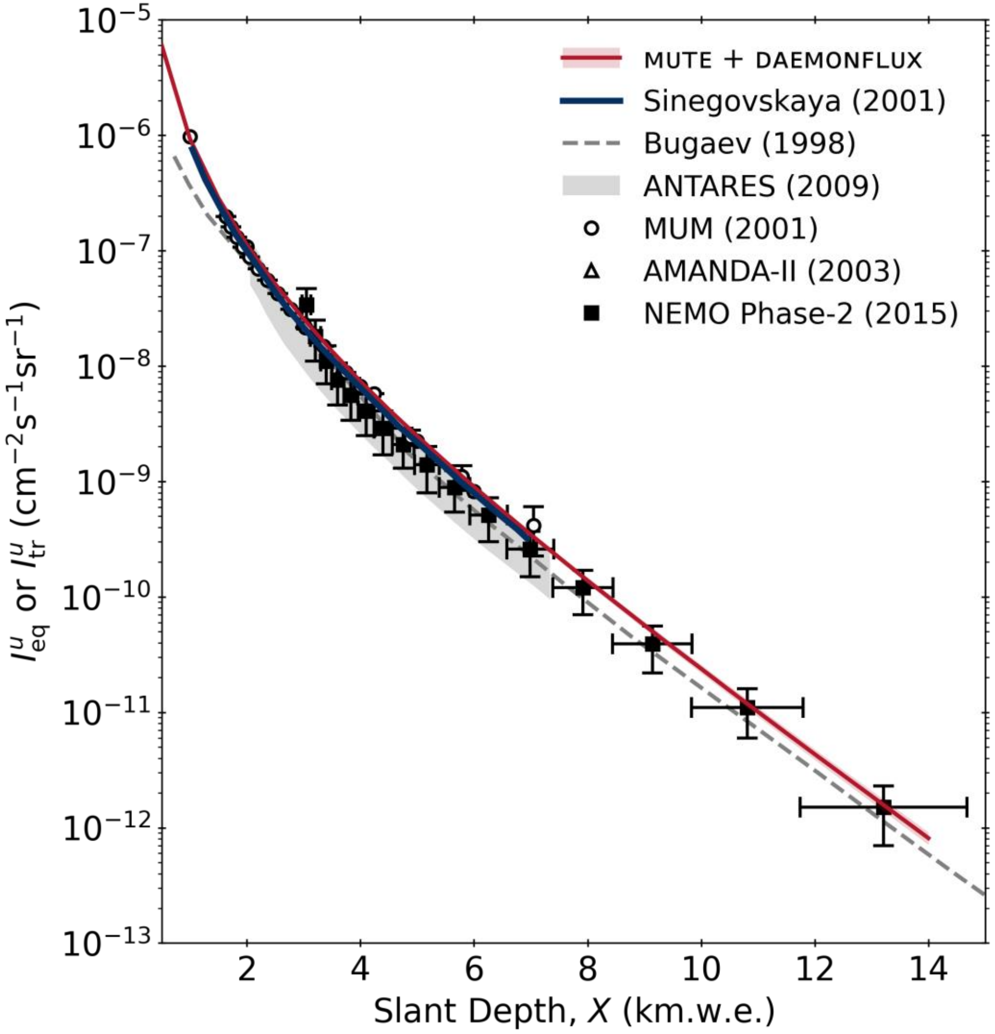
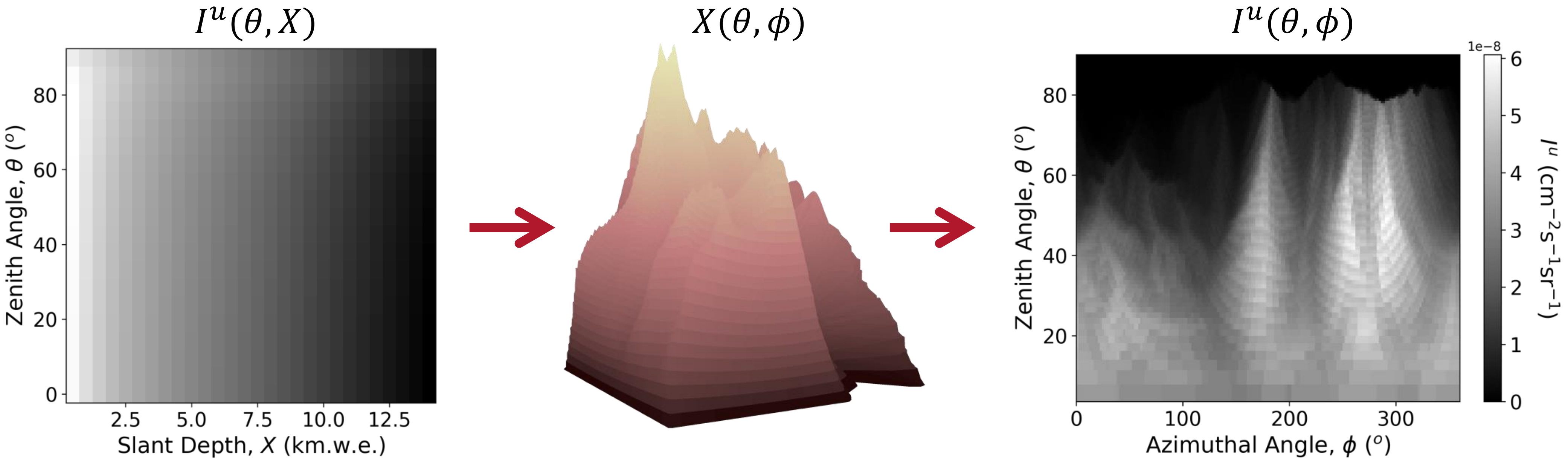


Underground Intensity for Mountains

- Underground intensities are first calculated on a grid of constant zenith angles and slant depths.
- Using a map of the mountain profile, these intensities are then interpolated to the slant depths $X(\theta,\ \phi)$ that define the mountain.



Underground Intensity for LVD

- MUTE calculates 2D underground muon intensities against θ and ϕ for labs under mountains.

- Data (raw counts and acceptance) was obtained from the LVD experiment at LNGS to compare to.

$$I^{u}_{\mathrm{LVD}}(\theta,\phi) = K\left(\frac{n(\theta,\phi)}{\varepsilon(\theta,\phi)}\right)$$

- K is a constant that provides physical units by requiring the total fluxes of the prediction and the data to be equal.

$$K = \frac{\Phi^{u}_{\text{tot, MUTE}}}{\iint_{\Omega}\left(\frac{n(\theta,\phi)}{\varepsilon(\theta,\phi)}\right)\mathrm{d}\Omega}$$

- Pulls show regions both in which the prediction is higher and lower than data, but no overall trend.

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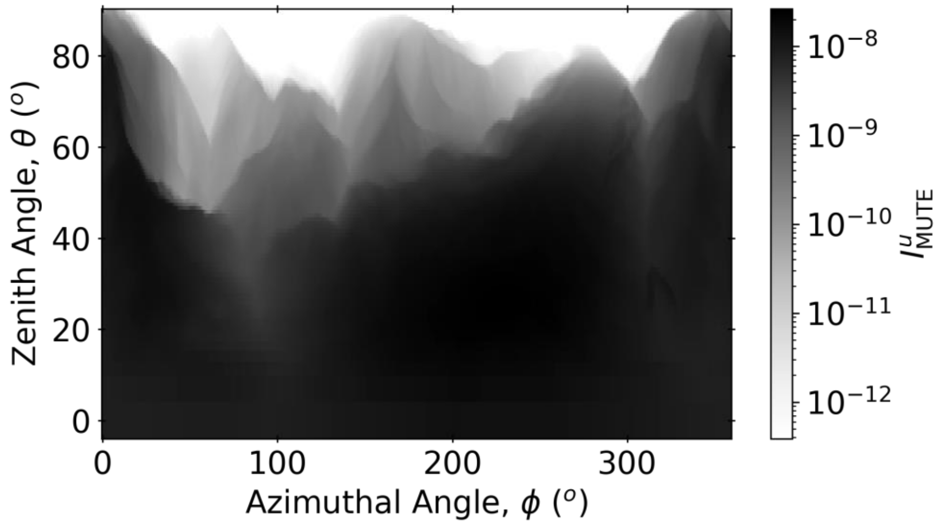
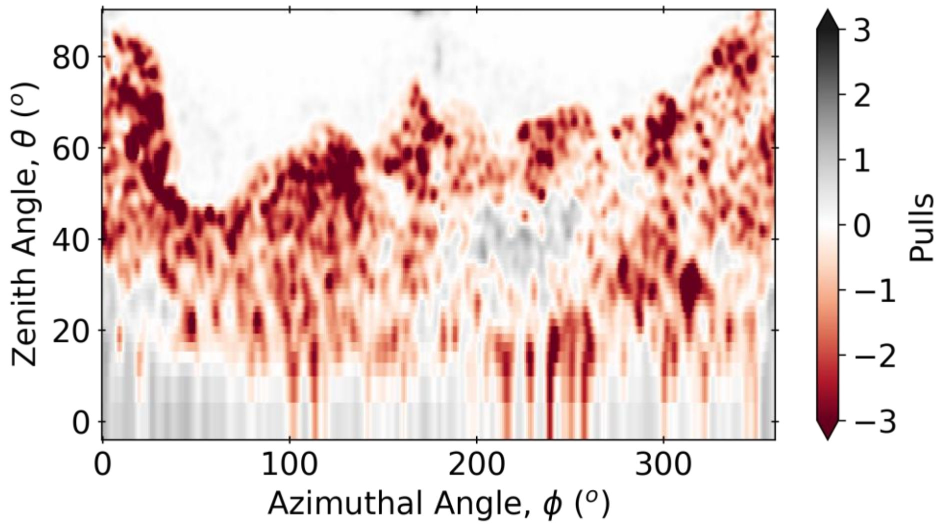




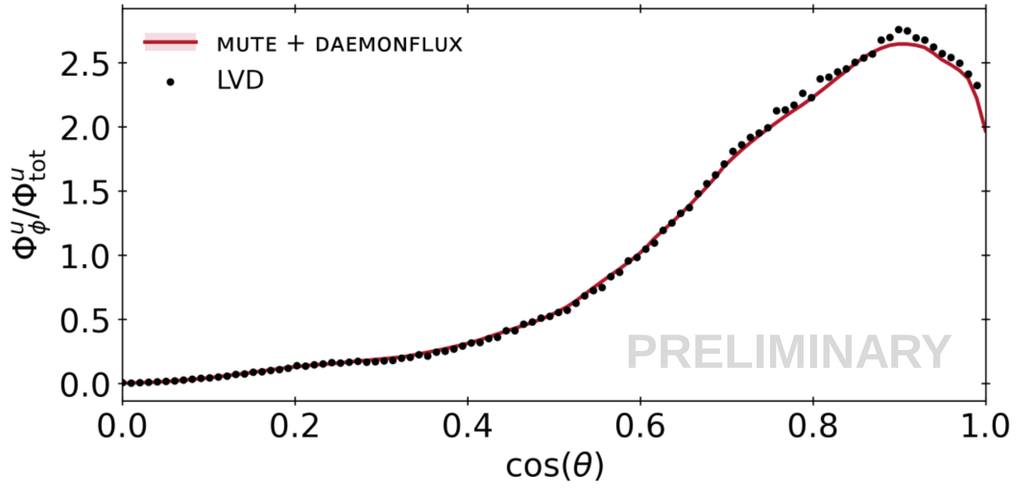
Underground Angular Distribution

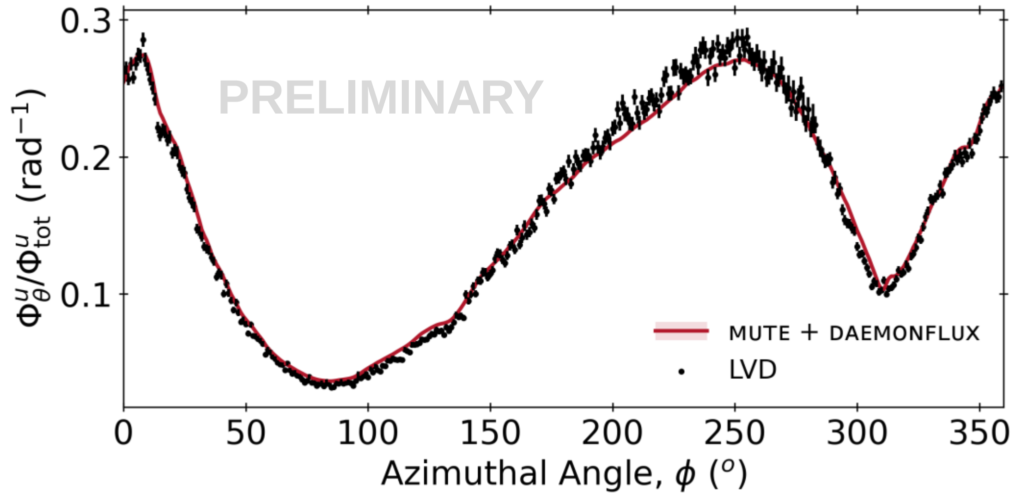
- One-dimensional projections of the intensity matrix give angular distributions in θ and ϕ.

$$\Phi^u_\phi = \int_0^{2\pi} I^u(\theta,\phi)\mathrm{d}\phi$$

$$\Phi^u_\theta = \int_0^{1} I^u(\theta,\phi)\mathrm{d}\cos(\theta)$$

- There is good agreement between the MUTE results and the LVD data.
- Further investigation suggests discrepancies are the result of an uncertainty in the mountain map not accounted for in the MUTE prediction.
- This can be used to cross-check data analyses:
  - Accuracy of mountain map
  - Event reconstruction





Underground Energy Spectrum

- MUTE integrates the flux over all angles to calculate the underground energy spectrum:

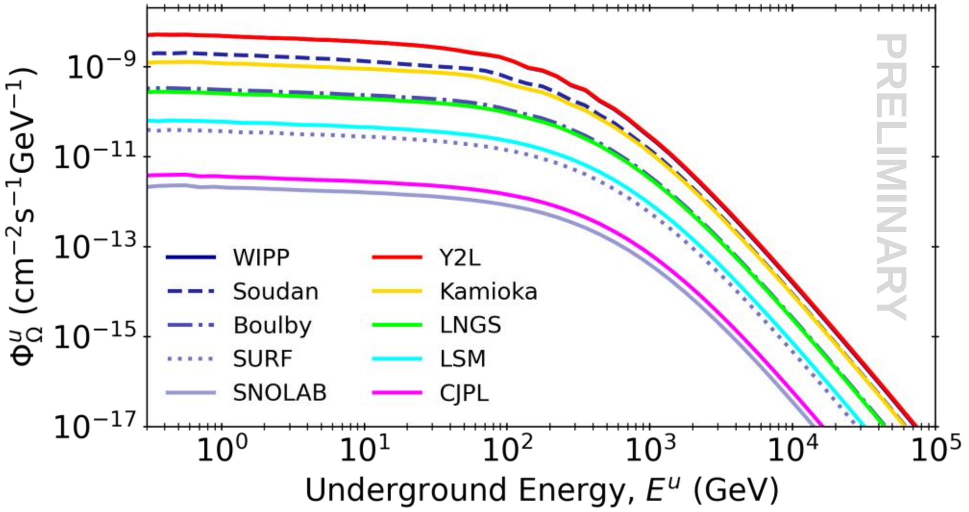
$$\Phi^u_\Omega(E^u, X) = \int_0^{2\pi} \int_0^1 \Phi^u(E^u, X, \theta) \, \mathrm{d}\cos(\theta) \, \mathrm{d}\phi$$

- Useful as input to particle transport codes for simulations of muon-induced backgrounds.

- The mean underground energy is calculated as the first moment of the energy spectrum:

$$\langle E^u(X) \rangle = \frac{\int_{E_{\mathrm{th}}}^{\infty} E^u \Phi^u_\Omega(E^u, X) \mathrm{d}E^u}{\int_{E_{\mathrm{th}}}^{\infty} \Phi^u_\Omega(E^u, X) \mathrm{d}E^u}$$

- Useful for calculating the ratio of through-going to stopping muons.



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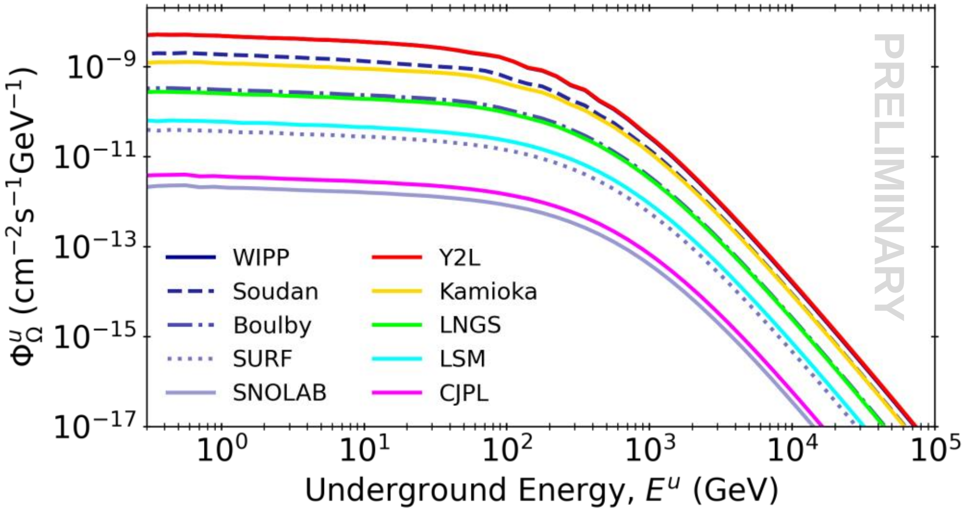
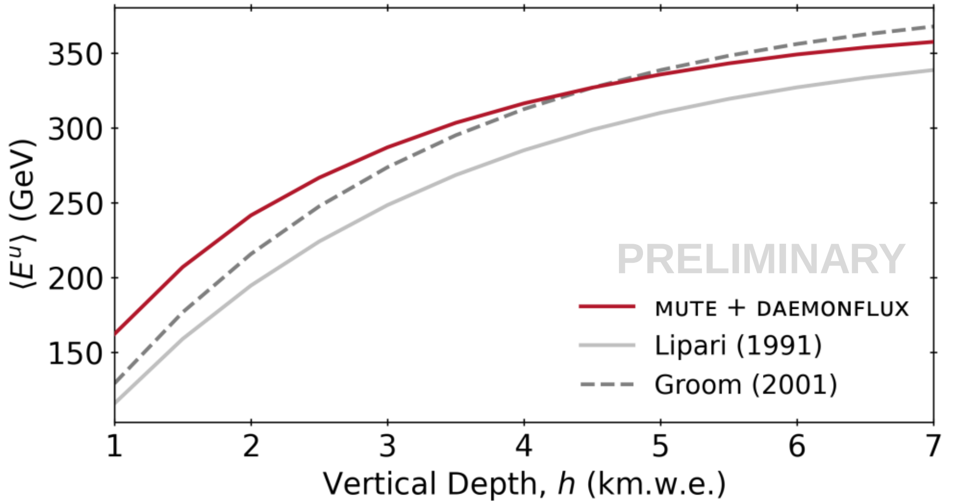
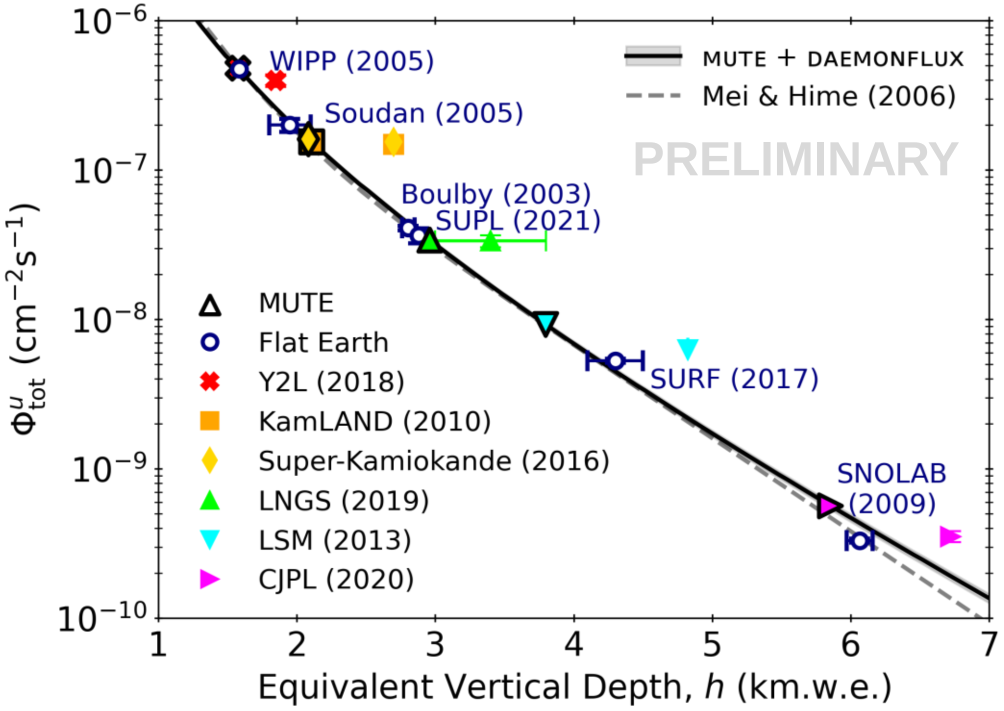
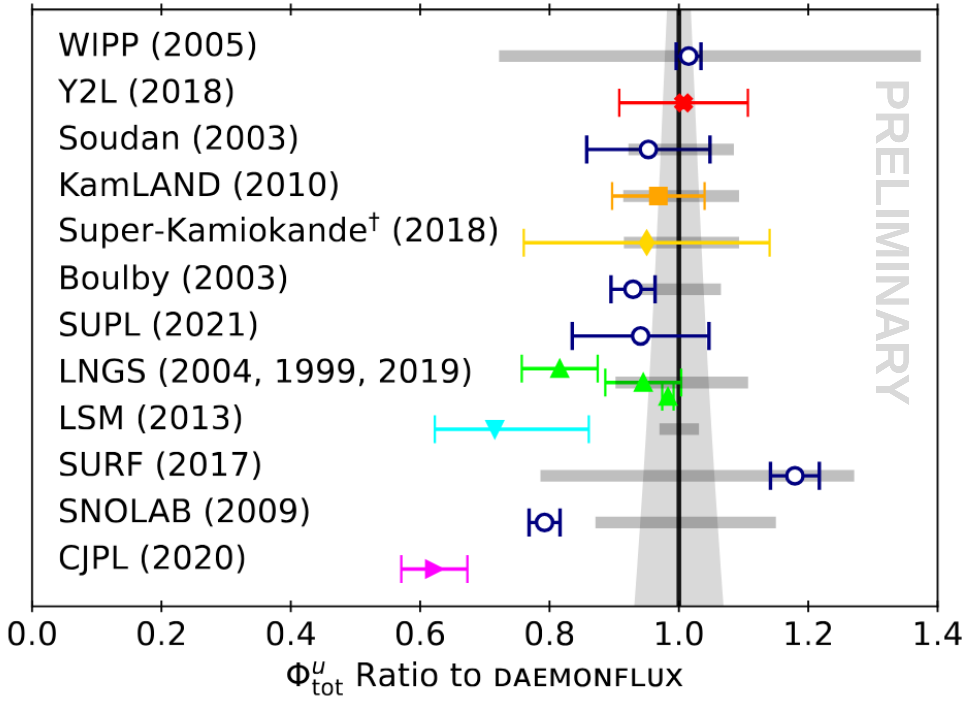




Total Underground Flux

- Total flux calculations are consistent with measurements for labs under flat overburdens and mountains within theoretical errors in most cases.





*** Plots updated since thesis**

Seasonal Variations

- Two physical effects contribute to the changes in muon production during summer:

Temperature increases

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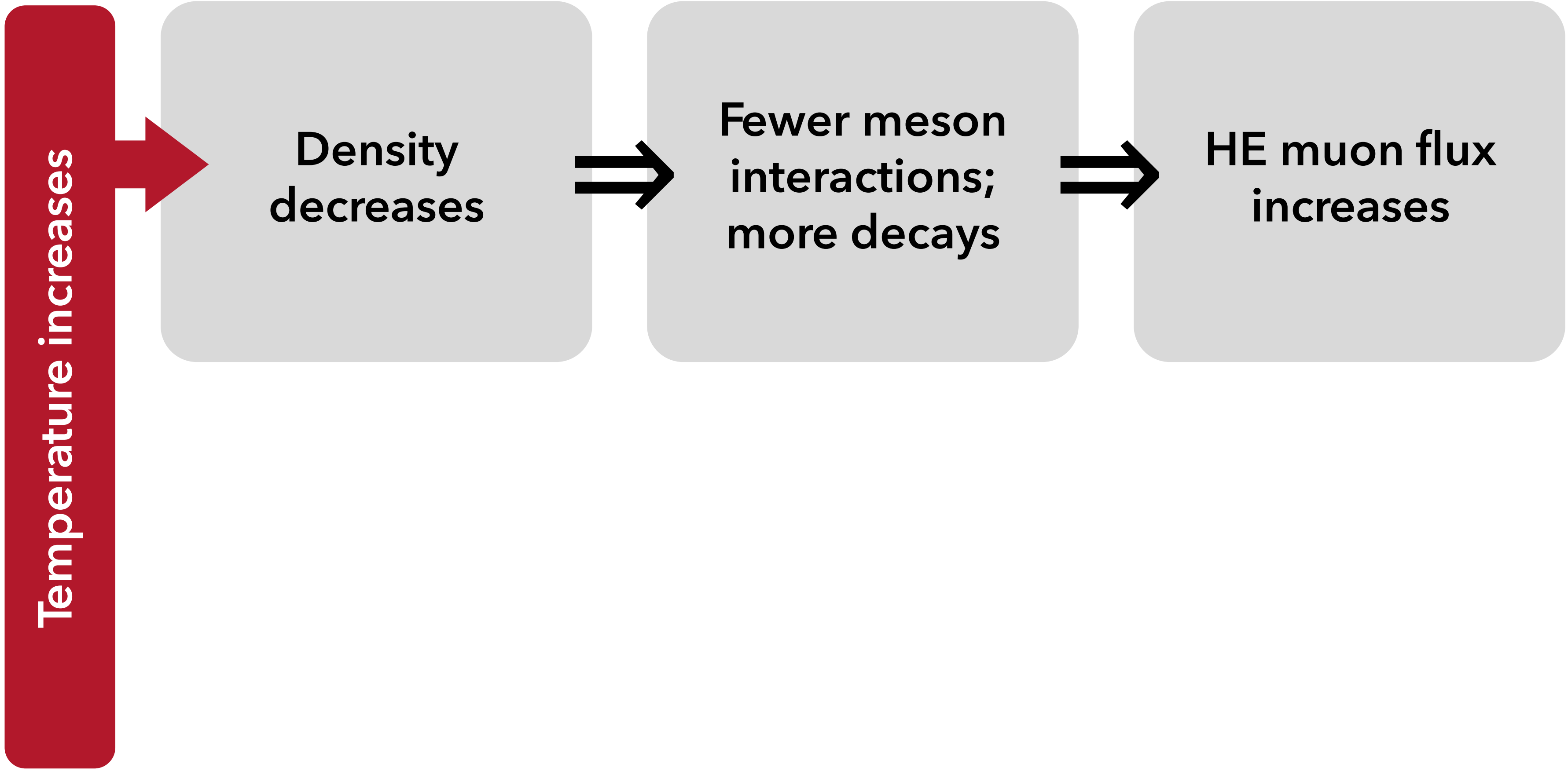

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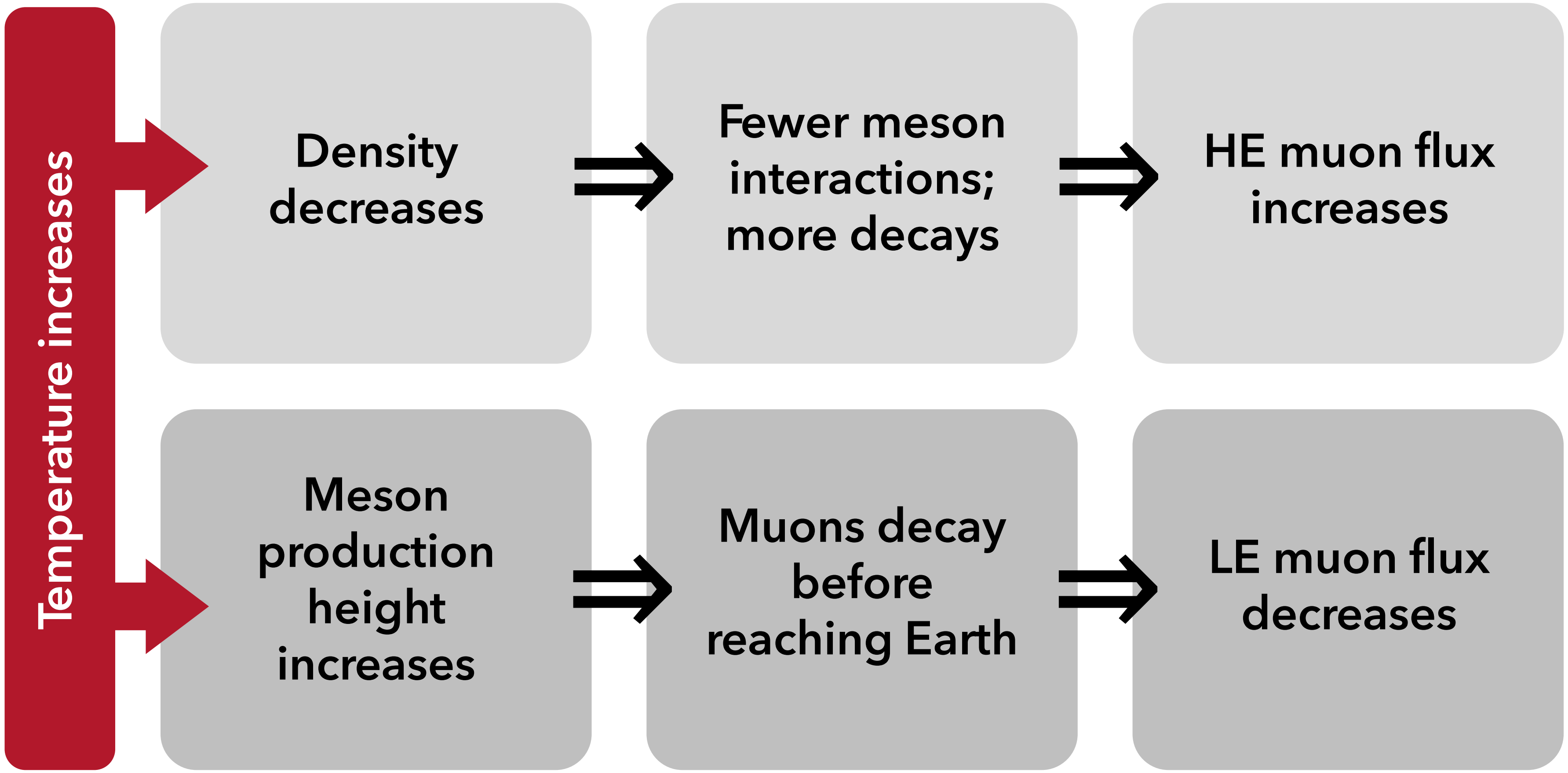

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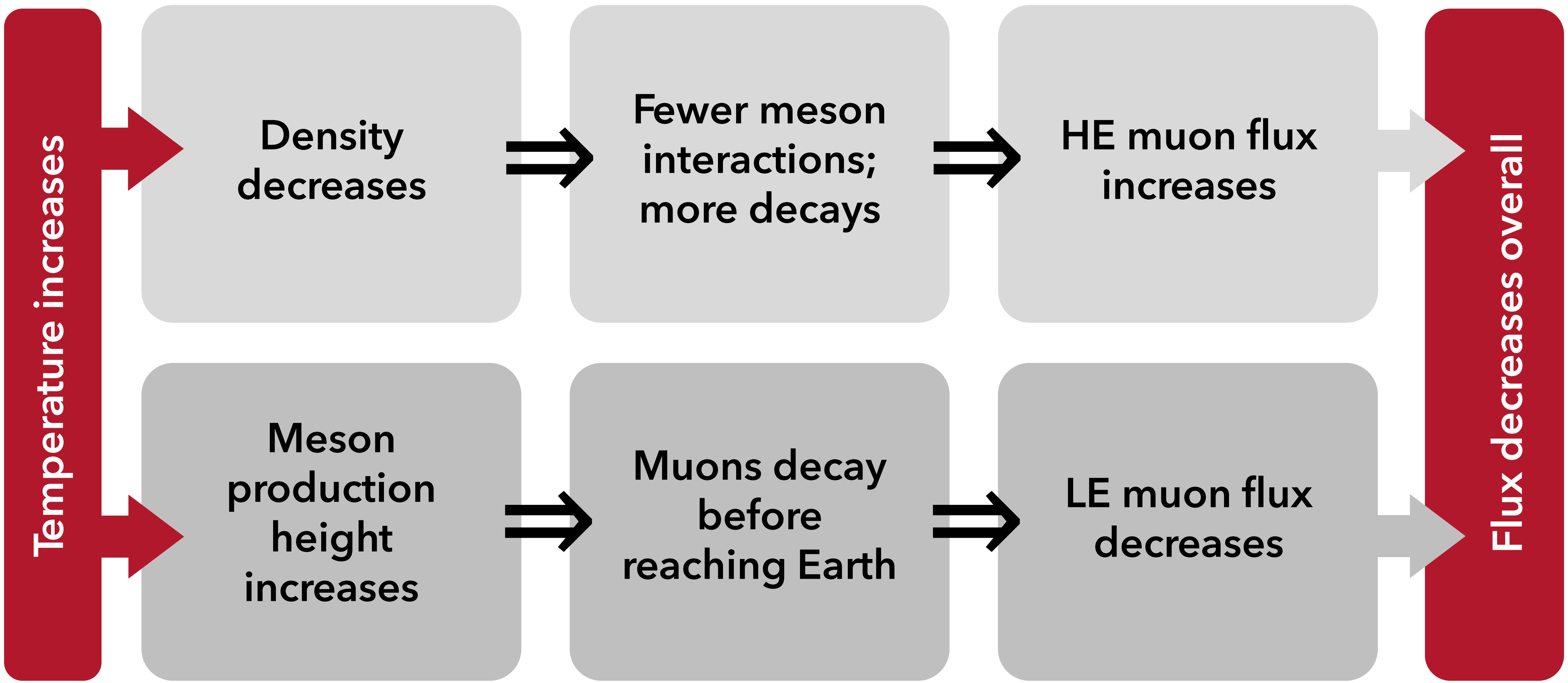
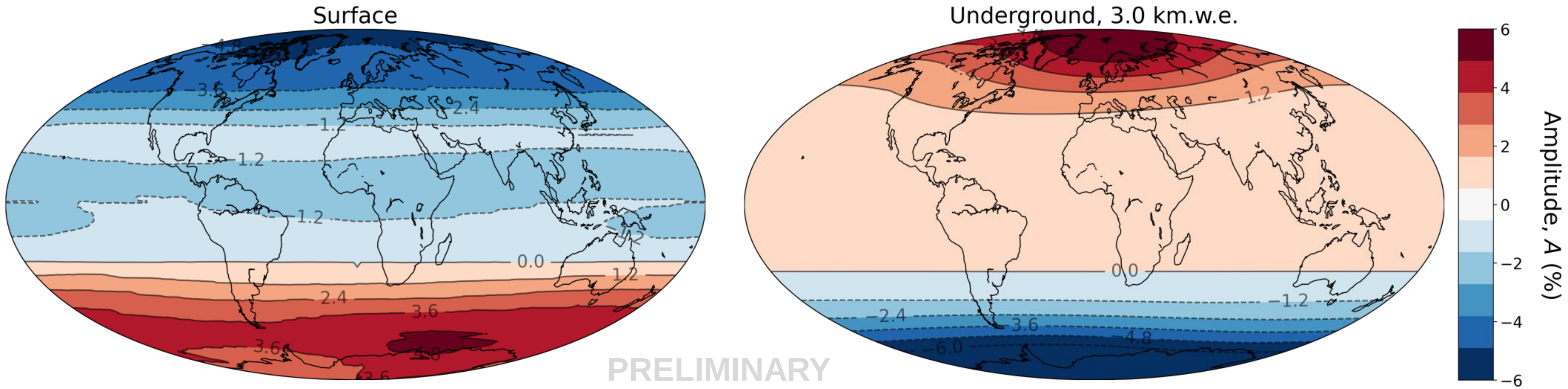


Seasonal Variations

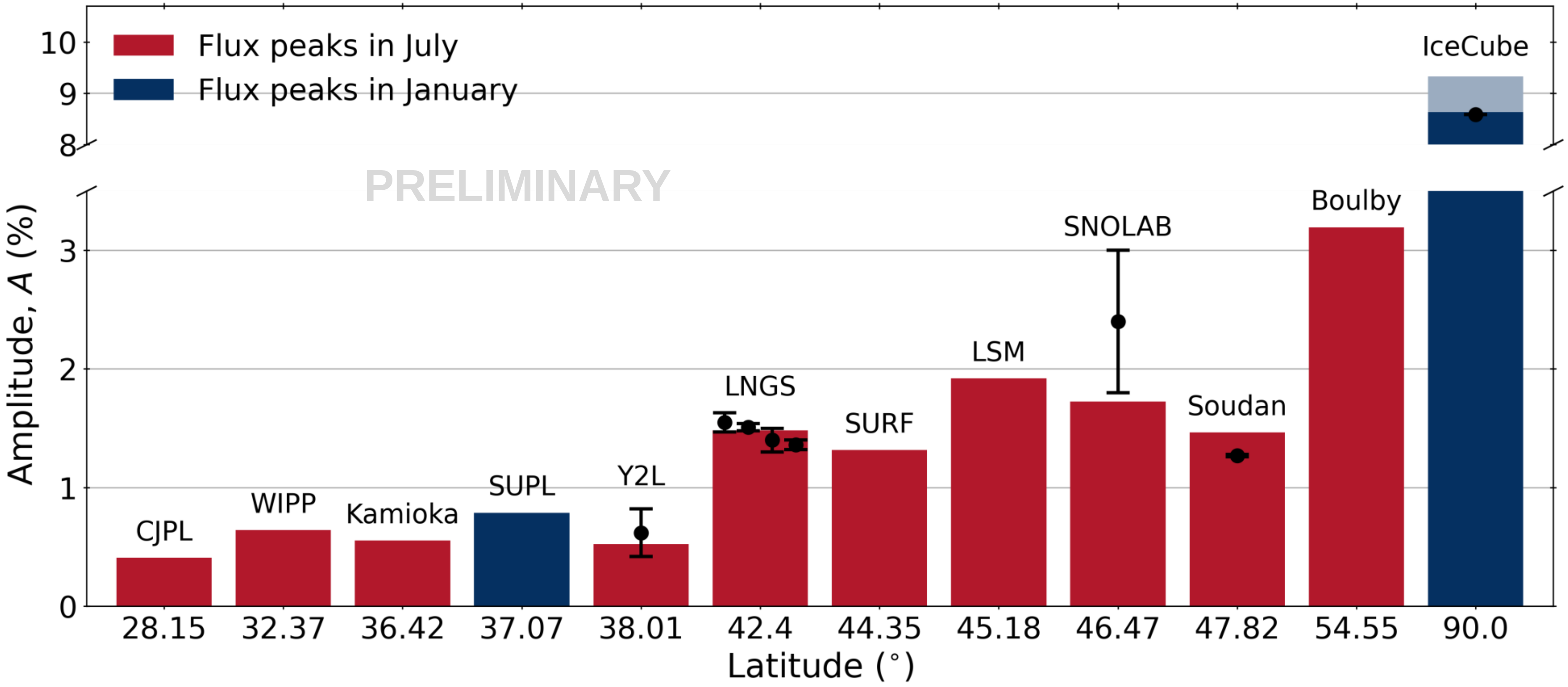
- The amplitude of the seasonal variations have been calculated around the globe:



- The muon flux is lower at the surface in summer in the northern hemisphere.
- However, there are more higher-energy muons in the summer, which reach deeper underground. Therefore, the muon flux is higher underground in summer.

Seasonal Variations

- MUTE can calculate seasonal variation amplitudes to high accuracy.



- Results will be submitted for publication soon.

Outlook

- Many further expansions and applications of MUTE are possible.

Release of v3.0.0

Upcoming release of newest version to include DAEMONFLUX and functions to calculate energy spectra and angular distributions.

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Interface MUTE with Geant4 or FLUKA so MUTE output can be used as input to these toolkits to calculate muon-induced neutron backgrounds for low-background experiments.

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Introduction | Method | Results | **Conclusion**

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Reducing Neutrino Flux Uncertainties

Because of the high precision of MUTE, use MUTE calculations to calibrate underground muon data to primary CR flux models to reduce uncertainties on atmospheric neutrino flux calculations.

Summary

- This research introduced MUTE, a new program to calculate atmospheric muon intensities and fluxes underground and underwater. MUTE improves on previous methods of calculating underground muons by performing data-independent forward predictions of muon fluxes to high precision.

- Results agree well with experimental data from various experiments around the world in almost all cases for vertical muon intensities, angular distributions, total fluxes, and seasonal variations of the total flux.

- Possibilities:

  - Publish double-differential intensity data instead of vertical-equivalent.
  - More measurements of energy spectrum and total flux.
  - Obtain more detailed knowledge of rock composition and density above lab.

- MUTE is offered as an open-source tool, and is flexible so experiments can tailor its use to their detector conditions.

- It is currently or will be used by a number of experiments:

  - NEWS-G, PICO, DEAP, P-ONE, nEXO, JNE, Super-Kamiokande, ALPHA-g

Thank you