Lab X: Hooke's Law

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Introduction

The purpose of this experiment was to measure the spring constant and mass of a spring.

Robert Hooke found the restoring force of a spring, $\underline{F}$, to be:

$$\underline{F} = -k\underline{x},$$

where k is the spring constant, and $\underline{x}$ is the displacement from equilibrium position. This is known as Hooke's Law. This experiment makes use of Hooke's Law in a mass-spring system. The oscillations of a mass hanging off a spring, when disturbed from its equilibrium position, are an example of simple harmonic motion, which is defined as a type of sinusoidal motion with a given constant period, T. For a mass-spring system, the period T can be related to the mass hung off the spring, M, by the following formula from the lab manual [1]:

$$T^2 = \frac{4\pi^2}{k}\left(M + \frac{m_s}{3}\right), \quad (1)$$

where m_s is the mass of the spring.

Experimental Method

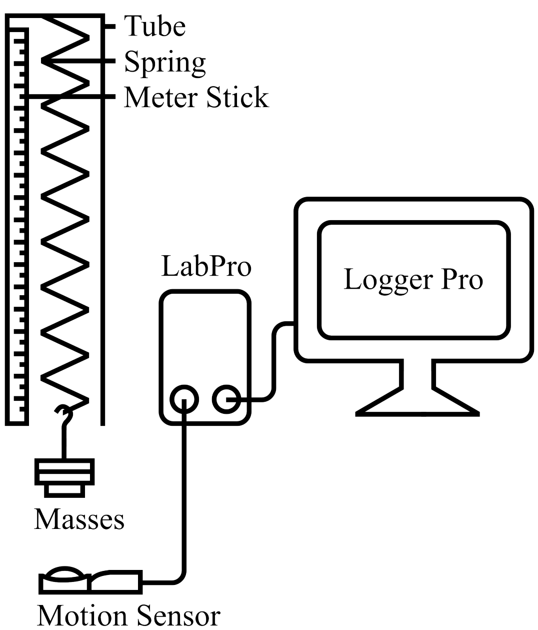

Figure 1: A diagram of the experimental set-up (personally drawn).

The experimental set-up consisted of a spring mounted inside a tube that contained a meter stick, as shown in Figure 1. The spring had a loop at the end, off of which a mass hanger and slotted masses were hung. A motion sensor connected to a Logger Pro LabPro unit was placed on the floor directly underneath the mass hanger in order to record position-time data of the bottom for the mass hanger during oscillations.

First, we hung a total mass of 0.250 kg (the sum of the 0.050 kg mass hanger and 0.200 kg slotted masses) from the loop in the spring. Reading from the meter stick inside the tube, we pulled the spring down approximately 2 cm, and then released it to begin the oscillatory motion. After the masses had completed two full oscillations, we clicked the 'Collect' button in Logger Pro to start the collection of the position-time data.

This was repeated for masses up to 0.450 kg, going up in 0.025 kg increments. After data was collected for all masses, a second trial was done.

Results

The motion sensor recorded data every 0.05 s for 5 s. Using this data, the period of oscillation, T, was calculated by finding the first two maxima in position, and subtracting their times, t_1 and t_2:

$$T = t_2 - t_1.$$

Two trials were done for each mass. The resulting values are shown in Table 1.

This data can be used to calculated the spring constant and the mass of the spring by linearising Equation (1). To put this in a linear form, rearrange to solve for M while isolating k:

$$M = k\left(\frac{T^2}{4\pi^2}\right) - \frac{m_s}{3}. \tag{2}$$

This is now in the form $y = mx + b$. In this case, plotting M vs $T^2/4\pi^2$ will result in a graph with a slope equal to the spring constant, and a y-intercept equal to $-m_s/3$. Values of $T^2/4\pi^2$ are also shown in Table 1.

Table 1: The mass and period data as extracted from Logger Pro measurements for both trials. M is the total mass that was hung from the spring (that of the mass hanger plus that of the slotted masses). Uncertainties in mass were assumed to be negligible. The period was calculated from the Logger Pro time-position data. Logger Pro took a measurement every 0.05 s, so the uncertainty in any given time is 0.05 s. The period was calculated from the difference of two times, t_1 and t_2, and so the uncertainty in period is $\sqrt{\delta t_1^2 + \delta t_2^2} = 0.07$ s. The error propagation for $T^2/4\pi^2$ is shown in Appendix A.

Mass, M / kg	**Period, T / s (± 0.07)**		$T^2/4\pi^2$ / s^2	
	Trial 1	**Trial 2**	**Trial 1**	**Trial 2**
0.250	1.16	1.18	0.034 (±0.004)	0.035 (±0.004)
0.275	1.21	1.21	0.037 (±0.004)	0.037 (±0.004)
0.300	1.25	1.25	0.040 (±0.004)	0.040 (±0.004)
0.325	1.29	1.29	0.042 (±0.005)	0.042 (±0.005)
0.350	1.33	1.32	0.044 (±0.005)	0.044 (±0.005)
0.375	1.36	1.36	0.047 (±0.005)	0.047 (±0.005)
0.400	1.39	1.39	0.049 (±0.005)	0.049 (±0.005)
0.425	1.43	1.43	0.052 (±0.005)	0.052 (±0.005)
0.450	1.46	1.46	0.054 (±0.005)	0.054 (±0.005)

Using Equation (2), the data in Table 1 was plotted in order to calculate the spring constant and the mass of the spring. This is shown in Figure 2.

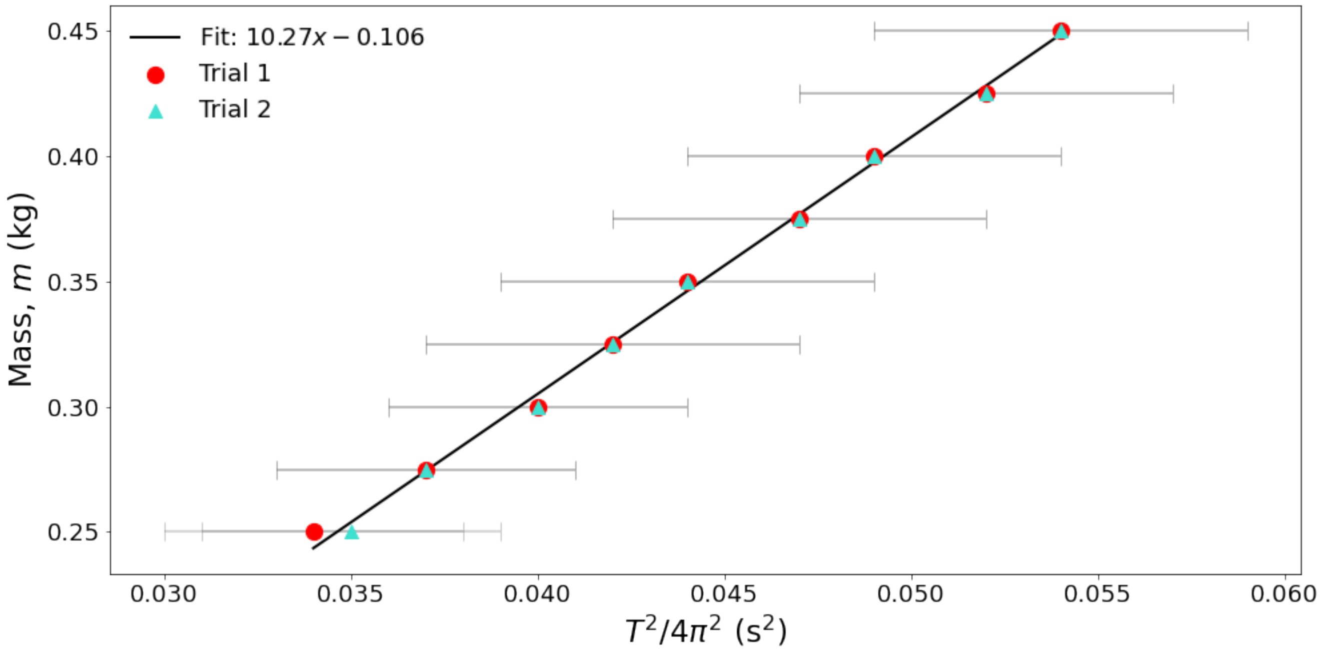

Figure 2: A plot of M vs $T^2/4\pi^2$ using the data for both trials in Table 1. Because many of the values between Trial 1 and Trial 2 are so similar, many points overlap. The slope of the line, which represents the spring constant, is (10.27 $\pm$ 0.08) Nm^{-1}, and the y-intercept, which represents negative one third of the mass of the spring, is (-0.106 $\pm$ 0.003) kg.

Using the `numpy.polyfit()` function, the slope and y-intercept of the graph in Figure 2 were calculated to be (10.27 $\pm$ 0.08) Nm^{-1} and (-0.106 $\pm$ 0.003) kg respectively. From Equation (2), the slope represents the spring constant, and the y-intercept represents negative one third of the mass of the spring. Therefore, to calculate the mass of the spring and its uncertainty, use:

$$m_s = -3b,$$

$$\delta m_s = 3\delta b,$$

where b is the y-intercept, and δb is the uncertainty in the y-intercept. The mass of the spring, therefore, is calculated to be (0.318 $\pm$ 0.009) kg.

Discussion

The graph in Figure 2 is linear, as expected from Equation (2). The spring constant was calculated to be (10.27 $\pm$ 0.08) Nm^{-1}, and the mass of the spring was calculated to be (0.318 $\pm$ 0.009) kg. Based on the spring used, both of these numbers seem reasonable [Note: No comparison is done here because there is no accepted or theoretical value to do a comparison with.].

Data from both Trial 1 and Trial 2 was plotted in Figure 2. Most of the data points overlap completely, but even those that do not do have error bars that overlap, and so all of the measurements taken in trial 1 are consistent with the measurements taken in trial 2. The purpose of taking two sets of data was to increase the precision of the linear fit. If only the trial 1 data had been used, the calculated spring constant would be (10.2 $\pm$ 0.1) Nm^{-1}. If only the trial 2 data had been used, it would be (10.37 $\pm$ 0.09) Nm^{-1}. Both of these have uncertainties greater than that of the spring constant calculated using both trials (0.08 Nm^{-1}), showing that, although not all of the trial

1 and trial 2 data points overlap exactly, they still help reduce the effect of random error in the measurements.

One possible source of error comes from friction between the spring and the edge of the bottom of the tube the spring was mounted in. Because the force of friction acts against the direction of motion, this friction would have caused the period to be larger than it would have been if the system were in vacuum. Therefore, due to the inverse relationship between T^2 and k in Equation (1), this would have decreased the value of k, whereas it would have increased the value of m_s.

Conclusion

The experiment was successful, as reasonable values were found for both the spring constant, k, and the mass of the spring, m_s.

Using a motion sensor and a Logger Pro LabPro unit, the positions of masses oscillating on a spring were recorded against time, from which periods of oscillation were calculated. Using these masses and periods, the relationship between T^2 and M for a mass oscillating on a spring with a non-negligible spring mass was plotted, and the linear slope and y-intercept gave the spring constant, k, of the spring to be (10.27 ± 0.08) $\mathrm{Nm^{-1}}$, and the mass of the spring, m_s, to be (0.318 ± 0.009) kg.

References

[1] Isaac, I., *et al.* (2021). *Lab Manual PHYS 144*. Edmonton: University of Alberta, Department of Physics.

[2] Harris, C. R., Millman, K. J., van der Walt, S. J., *et al.* (2021, June 22). *numpy.polyfit*. NumPy. https://numpy.org/doc/stable/reference/generated/numpy.polyfit.html

Acknowledgements

None.

Appendix A

To propagate errors in $T^2/4\pi^2$, use the general error propagation formula for functions of one variable, where the variable here is T:

$$\delta\left(\frac{T^2}{4\pi^2}\right) = \frac{\partial}{\partial T}\left(\frac{T^2}{4\pi^2}\right)\delta T.$$

Taking the derivative:

$$\delta\left(\frac{T^2}{4\pi^2}\right) = \left(\frac{1}{4\pi^2}\right) 2T\delta T.$$

The final equation for the uncertainty is:

$$\delta\left(\frac{T^2}{4\pi^2}\right) = \frac{T\delta T}{2\pi^2}.$$

This is the equation that was used in Table 1 to calculate uncertainty values for $T^2/4\pi^2$ for trials 1 and 2.